EXAMPLES

IN

FIRST YEAR PHYSICS

Pre-University Course

LAXMIDAS G. PATEL, B. A., B. SC.

THE STUDENTS' OWN BOOK-DEPOT

DHARWAR

1959

BY THE SAME AUTHOR

PRE-UNIVERSITY

For First Year Science

FirstYear Practical Physics

B. Sc. Part I

For Intermediate Science

Examples

IN

Intermediate Physics

Intermediate Practical Physics

Fully Solved

Examples in Physics

FOR

Intermediate Students

Publishers:

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by

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PREFACE

" Our pleasure in pursuing a subject in direct proportion to the degree of our mastery of it. Mastery of a subject involves the ability to use it. Solving numerical problems is one of the most effective means for developing the ability to make use of the principles of Physics."

—*E. S. Ferry.*

I, therefore, find it a great pleasure to offer in the following pages a list of problems designed to furnish experience in the application of the principles, which a First Year Science student of the Bombay University comes across in the study of his subject. It is acknowledged that there is a great need felt by the students for a collection of problems based on the Syllabus. The students have to refer to difficult books, which, however, are full of problems beyond their scope. This book is intended to supply this want by presenting to the students, problems which are just suited to their requirements.

The problems in this book consist for the most part of exercises usually framed for class-work during the course of the last twelve years. To these have been added questions from University Examinations and the source from which each question is taken is in every case acknowledged. In framing the problems, I have kept in view the modern applications of the principles of Physics to industry and every-day life and in arranging their sequence, I have followed the revised syllabus in First Year Science course of the Bombay University.

Some problems have been fully worked out to illustrate the plan of attack in arriving at the solutions. Hints are given for some of the difficult problems, showing the outline of the method and enabling the student to think and find out the solution for himself. This exercise of conscious reflection will obviate the mechanical application of the formulæ. By making intelligent use of various physical relations, the student will acquire skill and dexterity in solving problems of the types required of him in his examinations and will be in a better position as to understand and appreciate the subject and its applications.

As far as possible, I have made an attempt to give at least two problems of each particular type, and their total number has been so chosen as to leave enough of them for the student to do by himself, in addition to those that might be solved in the class

room, with one period a week on an average. Problems of an advanced type have been marked with an asterisk.

In the beginning of each section, I have given a list of some of the important equations used in the sections, and I have also taken care to use the symbols generally found in Text-books on Physics.

The subject matter of this book is directly taken from my two books,— Problems in First Year Physics and Examples in Physics for Intermediate students—, which have been largely appreciated by the student world during the course of last seven years. I have, however, greatly modified the plan of arrangement to suit the changes in the syllabus and also to enable the First Year students to follow and grasp the theory with greater ease.

Since the above mentioned books, now, form the foundation of this book, I feel it my duty to express my heartfelt thanks to Prof. V. B. DIVATIA. M. A., I. E. S. (now retired) of the Gujarat College, Ahmedabad, to Prof. R. N. JOSHI, B. sc., of Fergusson College, Poona, and to prof. T. K. DEOLALKAR, M.A., B.Sc., and Prof. N.G. MOHILE, M.A.,B.Sc., of the Karnatak College, Dharwar who had been kind enough to go through the manuscripts and offer valuable suggestions when the same were under prepration for the first time in 1920. I am also thankul to Mr. V. Y. JATHR, B. sc., for getting the book prited in a short time.

Suggestions for the improvement of the book will be most thankfully received

KARNATAK COLLGE, DHARWAR L. G. PATEL
March, 1937

Second Edition

Some of the portion has been removed from the text to meet the requirements of the syllabus revised this year and wherever necessary some corrections have been made in accordance with the suggestions received. A set of miscellaneous examples for revision has been added at the end.

My thanks are due to Professor N. N. Bhagawat and N. R. Muley for the suggestions.

DHARWAR. *Nov. 1940* L. G. PATEL

Sixth Revised Edition

Examples on the topics included in the new Syllabus of the Universities have been added and the book has been brought upto date. We are greatful to Prof. D. R. Naik for helping us in this matter. PUBLISHERS.

CONTENTS

EXAMPLES IN FIRST YEAR PHYSICS

(*Pre-University Course*)

UNITS AND MEASURING INSTRUMENTS

1. Units

1. State which of the following units are fundamental and derived ones:— (1) Gramme, (2) Poundal, (3) Erg, (4) Watt, (5) Hour, (6) Foot, (7) Centimetre, (8) Foot-pound, (9) Horse-power.

2. Make a separate list of absolute and gravitational units from the following :— (1) Erg, (2) G. Cm., (3) Dyne-cm., (4) Pound, (5) Poundal, (6) Foot-pound, (7) Horse-power, (8) Watt.

3. What are scalar and vector quantities ? Sort out scalar from the vector quantities from the following list :— Volume, force, density, work, surface tension, acceleration, speed, mass, weight, electric potential, momentum.

Hint : A scalar quantity is one with which no idea of direction is associated and is completely defined by its magnitude whereas a vector quantity has both magnitude and direction.

4. Obtain the dimensions of the following :—
(1) Work, (2) Strain, (3) Momentum, (4) Angular velocity, (5) Density, (6) Impulse.

(1) Work is defined as 'Force × Distance'; but Force is defined as 'Mass × Acceleration'; therefore the dimensions of Work are—

$$[\text{Work}] = M \times LT^{-2} \times L = [M]\,[L]^2\,[T]^{-2}.$$

i.e. 1 in mass, 2 in length and -2 in time. (Similarly try the other cases.)

5. Derive the dimensions of :—(1) Force, (2) Power, (3) Surface tension, (4) Volume, (5) Specific gravity.

6. Give the dimensions of the following quantities :—(1) Acceleration, (2) Pressure, (3) Young's modulus, (4) Force.

*7. Derive the relation between the joule and the foot-poundal. Use the method of dimensions.

1 lb. = 454 gm., 1 ft. = 30·5 cm.

$$\frac{1 \text{ ft. pdl.}}{1 \text{ erg.}} = \frac{ML^2T^{-2} \text{ in } F.P.S. \text{ units}}{ML^2T^{-2} \text{ in } C.G.S. \text{ units}}$$

$$= \frac{454}{1} \times \frac{(30{\cdot}5)^2}{(1)^2} \times \frac{(1)^{-2}}{(1)^{-2}} = 422 \times 10^3.$$

$$\therefore \ 1 \text{ ft.pdl.} = 422 \times 10^3 \text{ ergs}$$

$$= (422 \times 10^3/10^7 =)\ 4{\cdot}22 \times 10^{-2} \text{ joules.}$$

*8. Obtain the relation between the watt and the horse-power using dimensional method.

*9. Examine in the following relations the indices of t in $\frac{1}{2}\,gt$ in (i); of r^2 in (ii); and of T and m in (iii) and correct them.

---

*Problems marked with asterisks may be omitted for the first reading. They are of advanced types.

The following abbreviations are used in marking the sources from which the problems are taken ;—

F. E.—First Exam. in Engineering, Bombay University.
S. E.—Second ,, ,, ,, ,,
I. Sc.—Intermediate Science Exam. ,, ,,
I.Ag.— ,, Agriculture ,, ,, ,,
B. A.—Final B.A. (Pass or Hons.) ,, ,,
B.Sc.—Final B.Sc. Subsidiary ,, ,, ,,
I. Sc. Madras-Intermediade Science Exam., Madras University.
B. A. Madras—Final B.A. ,, ,, ,, ,,

(i) $s = vt + \frac{1}{2}gt$ — Where the space travelled by a body is in terms of velocity, acceleration and the time.

(ii) $T = \frac{r^2 h \rho g}{2}$ — Where the surface tension of a liquid is in terms of radius, height, density of the column of the liquid and g.

(iii) $v = \frac{T}{m}$ — Where the velocity of a transverse wave along a stretched string is in terms of the tension acting in the wire and the mass per unit length.

(i) Here $[L] = [LT^{-1}] \times [T] + [LT^{-2}] \times [T]$.

$\therefore \quad [L] = [L] + [LT^{-1}]$.

Dimensions on both the sides should be equal. Hence the right-hand side LT^{-1} should be L showing thereby that $\frac{1}{2}gt$ should be multiplied by t, thus the corrected relation is $s = vt + \frac{1}{2}gt^2$.

(iii) $LT^{-1} = \frac{MLT^{-2}}{ML^{-1}} = L^2T^{-2}$, which is inconsistent as both the sides must have the same dimensions, which can be by squaring the left-hand term or taking the square root of the right-hand side term. Hence the relation should be

$$v^2 = \frac{T}{m} \text{ or } v = \sqrt{\frac{T}{m}}.$$

N.B.— Item (ii) has been left unsolved an an exercise.

*10. Test the accuracy of the following relations using dimensions :— (1) $s = \frac{1}{2}gt$, (2) $s = vt^2$, (3) $v_t = v_0 + \alpha t^2$, (4) $Fs = \frac{1}{2}mv$ where the symbols have the usual meanings.

11. The period of a simple pendulum depends upon its length, mass, acceleration due to gravity and the angle of vibration. Find by the method of dimensions the form of equation connecting these quantities.

Since the angle of vibration θ has no dimensions we omit it, then we have

$$[\text{ Period }] \propto [\text{ Length }]^x \, [\text{ Mass }]^y \, [\text{ acceleration }]^z$$

$$\therefore \quad [T] = [L]^x [M]^y [LT^{-2}]^z$$

$$\therefore \quad T = L^{x+z} M^y T^{-2z}.$$

Equating Mass, $y = 0$, equating Length, $x+z = 0$, and equating Time, $-2z = 1$ we get $\therefore y=0$, $z= -\frac{1}{2}, x=\frac{1}{2}$, Hence period $\propto$ [Length] $\frac{1}{2}$ [acceleration] $-\frac{1}{2}$.

$$\therefore \quad \text{Period} \propto \sqrt{\frac{\text{Lnegth}}{\text{acc.}\, g}}.$$

12. Frequency of vibration of a sonometer string depends on the mass, the length of the vibrating string and on the tension applied to it. Determine the form of equation connecting these quantities. Use dimensions.

2. Callipers

13. A scale is divided into millimetres and a vernier scale is attached to it. 9 divisions on the main scale are equal to 10 divisions on the vernier scale. What is the least fraction of a centimentre that can be read by the vernier ?

14. A barometer scale is graduated in millimetres. 19 divisions on the main scale are divided into 20 on the vernier piece. What is the least count of the instrument ?

15. The scale of a travelling microscope is in half millimetres. What kind of vernier will give the least count of 0·01 mm. ?

16. A barometer has the main scale in $\frac{1}{20}$ of an inch. You are required to construct a vernier for it to read to 0·002 in. How will you do it ?

17. A circular scale reads to 10 minutes of an arc. How will you construct a vernier to enable you to read

to 10 seconds? If the main scale is in half degrees and 29 div. on it equal 30 div. on the vernier, state the least count.

18. A polarimeter has a circular scale divided into $\frac{1}{4}$ of a degree. The vernier has 25 div, equivalent to 24 div. on the main scale. What fraction of a degree can the instrument give?

19. Between the jaws of a vernier callipers is kept a block of such a length that line 6 of the vernier scale coincides with line 9 of the main scale, and consequently the zero of the vernier scale is a short distance to the right of line 6 of the main scale. If the main scale divisions are $\frac{1}{20}$ of an inch each, and 25 vernier divisions are equal to 24 main scale divisions, find the length of the block. State the sensitiveness of the callipers.

20. A barometer scale has a vernier giving the least count of $\frac{1}{500}$ in. The height of the column is found to be about 29·90 in division on the main scale with the 14th vernier line coinciding. What is the true height?

21. A vernier callipers is seen to give a reading of 3·7 cm. on the main scale and the 8th vernier division in coincidence. What is the length of the solid if the least count is $\frac{1}{100}$ cm.?

3. Micrometer Screw

22. The pitch of a certain micrometer screw is 0·05 in. and the screw head has 50 divisions. After setting upon a sphere and then removing it, 7 complete turns and 28 divisions are required to cause the screw to advance to the stop. Find the sensitiveness of the instrument. What is the volume of the sphere?

50 div. on the head are to be turned through to

move the screw 0·05 in. $\therefore$ 50 div. are equivalent to 0·05 in. Hence the *sensitiveness* = 0·001 in. Again 7 complete turns and 28 div. on the head give a measure of the diameter. Hence the diameter $= 7 \times 0{\cdot}05 + 0{\cdot}001 \times 28 = 0{\cdot}378$ in. The volume of the sphere is, therefore, $\frac{3}{4}\pi r^2 = 0{\cdot}0283$ cu. in.

23. In a micrometer screw-gauge, there are 10 threads of the screw per centimetre of its length, and 100 divisions are marked on the head. When adjusted without anything between the screw and the stud, the instrument shows the 5th division on the head coinciding with zero on the main scale. After setting upon a block and removing it, 10 complete turns and 55 divisions are required to cause the screw to advance to the stop. Find the thickness of the block.

Hint: Allow for the initial error of the apparatus. The error is *positive.**

24. The least count given by a micrometer screw is $\frac{1}{100}$ mm. The head scale is found to overwind by 5 div. beyond the zero of the pitch scale when the gap is closed. What is the zero correction? When a wire is held between the jaws, the reading on the pitch scale is found to be 2 mm. and that on the head scale 18 div. Calculate the diameter of the wire.

25. The pitch of a screw is $\frac{1}{2}$ mm. and the divisions on the head scale are 50. When the gap is closed, the head scale zero stops short of the guide line by 5 div. When a glass plate is held in the gap, the reading on the pitch scale is found to be 3 mm. while the 55th div. on the

---

* The error is +*ve*, if it is above the marked zero of the scale and −*ve* if below that. Errors are always to be subtracted from the subsequent readings.

head scale is seen along the guide line. Determine the thickness of the glass plate.

4. Spherometers

26. Find the radius of curvature of a spherical surface from the following readings taken with a spherometer :—

(1) When adjusted on a curved surface.. 0·029 cm.
(2) When adjusted on a plane surface... – 0·003 cm.
(3) The distance between the screw-end and one of the legs............................2·4 cm.

[I Sc. 1923]

Here sagitta $h = 0{\cdot}029 - (-0{\cdot}003) = 0{\cdot}032$ cm. and $a = 2{\cdot}4$ cm.; hence using the usual relation,

$$R = \frac{a^2}{2h} + \frac{h}{2} \quad \therefore \text{ we get } R = \frac{2{\cdot}4^2}{2 \times 0{\cdot}032} + \frac{0{\cdot}032}{\cdots}.$$

i. e. radius of curvature = 90·016 cm.

27. The following are the readings taken with a spherometer :—

(1) Number of threads per centimetre of the screw 20
(2) Number of divisions on the circular head 100
(3) Reading on the disc, when adjusted on a plane surface................................ + 1·5 div.
(4) Reading when adjusted on the curved surface ....81·5 div.
(5) Distance between any two outer lege... 4 cm.

Calculate the radius of curvature of the surface.

Hint : Use the relation $a^2/6h + h/2$ here, where $a = 4$ cm.

28. Find the radius of curvature of a given concave surface from the following data obtained with a spherometer.

(1) Pitch of the screw .... 0·5 mm·

(2) Divisions on the circular disc............100
(3) Distance between any two outer lege.. 4.0 cm.
(4) Reading on the disc, when adjusted on a plane surface .... + 18 div.
(5) Reading on the disc, when adjusted on the spherical surface ....–1 mm. & 68 div.

Hint : Reading for the spherical surface are below the zero on the main scale. The value of $h = 1$ mm. and $18 + (100 - 68)$ div. $= 1{\cdot}25$ mm.

29. A spheromet r reads 0·465 mm. when adjusted on a plane surface and 2·415 mm. when on a spherical surface. The distance between any two outer legs is 4 cm. Find the radius of curvature of the surface.

30. Find the thickness of a glass plate for which the following readings were taken with a spherometer. Pitch is $\frac{1}{2}$ mm.. Divisions on the head scale are 100. When adjusted on a plane glass it reads 2 div. on the head scale below the zero of the pitch scale and when adjusted on the given plate, 48 div. on the head scale above the pitch scale zero.

31. If an error of 1% is made in measuring (i) h (ii) a in the experiment to determine the radius of curvature of the convex surface of a plano-convex lens using a spherometer, by how much will the value of L be wrong in each case when $h = 0{\cdot}1$ cm., $a = 4$ cm.

PROPERTIES OF MATTER

1. Density

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$D = \frac{M}{V}$$

$$\left.\begin{array}{r}\text{Sp. gr. } \textit{or} \\ \text{relative density}\end{array}\right\} = \frac{\text{Density of a body}}{\text{Density of water}}$$

$$= \frac{\text{Mass of a body}}{\text{Mass of water}} \text{ taking equal volumes}$$

$$= \frac{\text{Wt. of a body}}{\text{Wt. of water}} \quad \text{"} \quad \text{"} \quad \text{"}$$

1. A piece of wood measures 4 cm. $\times$ 20 cm. $\times$ 8 cm. and weighs 400 gm. Determine its density.

The volume of the piece $= (4 \times 20 \times 8 =)$ 640 cc. while the mass of the piece $=$ 400 gm.

As the density is mass per unit volume we have, here, density of the wood $= (\frac{400}{640} =)$ 0·625 gm. per cc.

2. A sphere of radius 3 cm. weighs 954·0 gm. Find its density.

3. A nugget of pure silver is 21 gm. If the density of silver is 10·5 gm./cc., determine its volume in cubic centimetres.

4. A swimming bath has a surface area of 40 yds. $\times$ 20 yds. and is 5 ft. deep on an average. How many gallons of water are required to fill it, if 1 cu.ft. of water weighs 62·5 lbs. ? 1 gallon of water weighs 10 lb.

5. A mass of 1600 lb. occupies a volume of 20 cu.ft. What is its density (i) in lb. per cu.ft. (ii) in gm. per cc. ?

(i) Since $D = \frac{M}{V}$, we get $D = \frac{1600}{20}$ *i.e.* 80 lb./cu.ft.

(ii) 1600 lb. = ($1600 \times 453{\cdot}6$) gm.
20 cu. ft. = (20×28315) cc.

$$\therefore \quad D = \frac{1600 \times 453{\cdot}6}{20 \times 28315}, = 1{\cdot}28 \text{ gm,/cc.}$$

6. Find the diameter of a metal sphere, if its mass is 792 gm. and sp.gr. 7·0.*

7. Calculate the mass of (a) a brass wire 50 cm. long and 7 mm. in radius, (b) a brass sphere 3 cm. in radius. The density of brass is 8·4 gm./cc.

8. Find in grammes the weight of a rectangular piece of a metal having its width = 4 in., breadth = 2 in., and thickness = 0·02 in. Sp. gr. of the metal is 8·4

9. A capillary tube is partly filled with mercury. The length of the mercury thread is 7 cm. The weight of the mercury in the tube is found to be 6·8 gm. Determine the radius of the bore of the tube. Sp. gr. of mercury is 13·6.

Since sp. gr. of mercury is 13·6
the density ,, is $13{\cdot}6 \times 1 = 13{\cdot}6$ gm./cc.
Again let r be the radius of the bore,
then the volume of the thread = ($\frac{22}{7} \times r^2 \times 7$) cc.

But, from the mass given, vol. $= \frac{6{\cdot}8}{13{\cdot}6} = 0{\cdot}5$ cc.

$\therefore \quad 22r^2 = 0{\cdot}5$. Hence $r = 0{\cdot}1507$ cm.

10. A narrow glass tubing weighed 14·20 gm. A thread of mercury 10·5 cm. long was drawn into it and weighed.

---

* Sp. gr. is a ratio of the same units, hence it is a mere number. It is the same in both the systems of units, The density differs in different systems as it is a mass contained in a unit volume.

The weight was found to be 18·73 gm. Calculate the diameter of the bore of the tube, if the sp.gr. of mercury is 13·59.

11. A flask when empty weighs 240 gm., when full of water 1340 gm. and when full of air weighs 241·3 gm. Find the density of air.

12. A sp.gr. bottle when empty, weighed 14·975 gm. When full of water, it weighed 39·875 gm., while full of a certain liquid, its weight was found to be 35·367 gm. Determine the sp. gr. of the liquid.

13. A sp.gr. bottle with water weighs 50 gm. and the same with water and 18 gm. of a powder in it weighs 57·5 gm. Determine the sp.gr. of the powder.

Wt. of the bottle full of water and the wt. of powder together = (50 + 18 =) 68 gm. But wt. of bottle containing both powder and water together = 57·5 gm.

∴ wt. of water having the same vol. as that of the powder = (68 – 57·5 =) 10·5 gm. But the weight of the powder in air = 18 gm. ∴ Sp. gr. of the powder = (18/10·5 =) 1·71.

14. A sp. gr. bottle weighs 72·84 gm. when full of water and when 20 lead shots weighing 6·09 gm. are introduced, it weighs 78·39 gm. Find the density of lead and the average diameter of each shot.

15. Following are the observations taken for determining the density of a soluble powder :—

Weight of sp gr. bottle empty	= 22·03	gm. wt.
,, ,, ,, ,, + powder	= 23·87	,,
,, ,, ,, ,, + ,, + kerosene	= 64·65	,,
,, ,, ,, ,, +kerosene alone	= 63·375	,,
,, ,, ,, ,, +water alone	= 72·84	,,

Calculate the sp. gr. of the powder.

Hint: First determine the sp. gr. of kerosene as in example 12 and the sp. gr. of the powder with respect to kerosene. Multiply this by the sp. gr. of kerosene.

DENSITIES OF MIXTURES AND ALLOYS

When there is no chemical change :—

Suppose $M_1, M_2, M_3, ..$ the masses of the substances $A_1, A_2, A_3, ..$ are mixed together so as to form a mixture (or an alloy), then if, V_1 V_2 V_3, –, be the respective original volumes, the density of the mixture (or the alloy)

$$= \frac{M_1+M_2+M_3+\cdots}{V_1+V_2+V_3+\cdots}$$

16. A litre of milk and a litre of water are mixed together. Determine the density of the mixture, if that of the milk is 1·63 gm./cc.

The mass of milk = $1{\cdot}03 \times 1000$ cc. $(D \times V = M)$

,, ,, of water = $1{\cdot}0 \times 1000$ cc.

The total volume of the mixture = $(1000+1000)$ cc.

$$\therefore \quad \text{density} = \frac{1030+1000}{1000+1000} = 1{\cdot}015 \text{ gm./cc.}$$

17. The densities of three liquids are as $1:2:3$. Equal volumes of the three are mixed together. Compare the density of the resulting mixture with that of the first liquid.

18. Three liquids A, B and C are mixed together. The volume and the sp. gr. of each are given in the following table :—

Liquid	Volume in cc.	sp. gr.
A	24	0·92
B	16	0·78
C	20	0·88

Find the sp. gr. when no change in volume takes place.

19. 20 cc. of a liquid are mixed with 20 gm. of water. If the reduction in volume of the mixture is 10% and the density of the liquid 0·80 gm./cc,, determine ths density of the mixture.

20. Find the density of a mixture of 200 cc. of water and 800 cc. of a salt-solution of density 1·2 gm./cc., assuming no chemical change.

21. In a piece of silver coin, the percentages of silver and copper are as follows :—

Silver 92·50% Copper 7·50%

If the densities of copper and silver are 8·93 and 10·50 gm./cc. respectively, determine the density of the coin.

Hint: Assume the mass to be 100 gm. Find the volume of each metal from the given percentage of masses that gives the total volume of the coin. The total mass having been assumed as 10 gm., find the density,

22. An impure lump of gold containing copper has $91\frac{2}{3}$%, of pure gold and $8\frac{1}{3}$% of copper in it. The density of gold is 19·32 gm./cc. and that of copper 8·93 gm./cc. Find the density of the lump.

23. A gold ring weighs 20 gm. and has a density 18 gm./cc. How much copper does the ring contain, if the sp. gr. of gold be 19·32 and that of copper in it 8·93 ?

Let x gm. be the mass of copper in the ring then $(20-x)$ gm. will be mass of gold in it.

But the density of pure gold is $(19{\cdot}32 \times 1 =)$ 19.32 gm./cc.

and ,, copper is $(8{\cdot}93 \times 1 =)$ 8·93 gm./cc.

$\therefore$ the vol. of copper $= \dfrac{x}{8{\cdot}93}$ cc. and vol. of gold $= \dfrac{20-x}{19{\cdot}32}$ cc.

Hence the total vol. of the ring $= \left(\frac{x}{8\cdot93}+\frac{20-x}{19\cdot32}\right)$ cc.

$\therefore$ its mass is $\left(\frac{x}{8\cdot93}+\frac{20-x}{19\cdot32}\right)18=20$ gm. $\therefore$ $x=1\cdot26$ gm.

24. A specimen of brass, which is an alloy of copper and zinc, weighs 100 gm. and has a density of 8·40 gm./cc. The densities of copper and zinc are 8·93 and 7·1 gm./cc. respectively. Find the proportion of the metals in the specimen.

25. The density of a specimen of market milk is found to be 1·02 gm./cc. If the sp. gr. of pure milk is 1·03 determine the volume of water in 3 litres of the market milk.

26. A 50 cc. flask filled with oil (sp. gr. 0·8) and water weighs 69·53 gm. Determine the amounts of water and oil in the flask. The empty flask weighs 22·03 gm.

2. Liquids-Pressure

The pressure at any point in a liquid is due to its weight and is directly proportional to the average depth h of the point below the free surface and to the density d of the liquid.

Hence force $F = A\times h\times d\times g$ (in absolute units)

and pressure $P=1\times h\times d\times g$ *i.e.* force per unit area.

27. A box, having a bottom of dimensions 20 cm. by 10 cm. and a depth of 10 cm. is filled with a liquid of density 1·2 gm./cc. Find the total force and the pressure acting on the bottom.

The area of the bottom $= (20\times10)$ 200 sq. cm.

$\therefore$ the total force $= 200\times10\times1\cdot2\,g.$ $(F = A\cdot h\cdot d\cdot g.)$

$= 2352000$ dynes.

The pressure $=\frac{2352000}{200} = 11760$ dynes/sq. cm. $\left(P=\frac{F}{A}\right)$

28. Total thrust on a surface of 10 sq. in. area is found to be 11·023 lb. wt. Find the average pressure (i) in lb.wt per sq. in. (ii) in. gm. wt. per sq. cm.

29. Find in gravitational units the thrust on the base of a vessel full of a liquid (sp. gr. 0·80) in the form of a trunketed cone. The area of the base is 20 sq. cm, and the height of the liquid column 30 cm. What is the pressure in dynes at a point in the base ?

30. A conical tumbler of glass is filled with glycerine (sp. gr. 1·26) to a height of 10 cm. from the bottom. What is the pressure in gravitational units at the bottom due to the liquid ?

31. A cylindrical vessel of base-area 24 sq. cm. is filled with mercury* to a height of 10 cm. and with water 7 cm. above that. Find the pressure on the bottom exerted by the liquids.

32. Find the depth of water in a lake, if the pressure due to it at the bottom is equal to that of a column of mercury 152 cm. long.

33. A stand pipe, 10 ft. high and 62 ft. above the ground level, is filled with water. Determine the force per sq. in, at the ground level.

Note :— A stand pipe, has generally its top and bottom of a conical shape. Here its bottom end is 62 ft. above the ground level.

34. The pressure gauge at a water tap in a theatre reads 41·66 lb./sq. in. Find the height of water in the reservoir above the level of the tap.

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* In all problems whenever required take,

the density of water in *F. P. S.* as 62·5 lb./cu. ft.

,, ,, in *C. G. S.* as 1 gm./cc.

,, mercury in ,, as 13·6 gm./cc.

g = 980 cm./sec.^{2} in *C. G. S.*

and = 32 ft./sec.^{2} in *F. P. S.* unless stated to the contrary.

The volume of the column of water exerting the given pressure is $1 \times h \times \frac{1}{144}$ cu.ft. where h is the height in feet.

But it is also $= \dfrac{41 \cdot 66}{62 \cdot 5}$ cu.ft. $\therefore \quad 1 \times h \times \dfrac{1}{144} \fallingdotseq \dfrac{41 \cdot 66}{62 \cdot 5}$

$\therefore \qquad h = 96$ ft.

35. The pressure at a water-tap at the base of a building is 42·5 lb. wt./sq. in. and on its top, it is 30 lb. wt./sq. in. Find the height of the building.

36. Find the force required to hold a board tightly against a hole 5 cm. long and 2 cm. broad in the side of a ship in a lake, if the average depth of the hole is 10 m. below the surface of water.

The force due to the column of water.

$= \text{Area} \times \text{average depth} \times \text{density} \times g.$

$= 5 \times 2 \times 1000 \times 1 \times 980.$

$= 9{\cdot}8 \times 10^6$ dynes.

$\therefore$ the force required to be applied in the opposite direction $= 10^4$ gm. wt. *or* $9{\cdot}8 \times 10^6$ dynes.

37. A cube, having a side of 20 cm. is suspended in water with its sides vertical and its upper edge 20 cm. below the surface of the liquid. Determine the average pressure on each of its faces.

Hint : The pressure on the top and bottom faces are given by the weight of the liquid column of unit cross-section and of height above the respective faces upto the surface of the liquid. The average pressure on the side faces will be given by the product of the depth at the centre of area of the face and the density of the liquid. *i.e.* by $(20+10) \times 1$.

38 A cubical vessel of side 20 cm. is closed at the top with an opening of area 10 sq. cm. in it. A vertical hollow tube of height 20 cm. is attached to the opening.

The vessel with the tube is filled with water. Find the force on each of the side of the vessel.

Area of a side is (20×20) 400 sq. cm.

Area of pressed part of the top is $(400-10=)$ 390 sq.cm.

Mean depth of the base is $(20+20=)$ 40 cm.

„ „ top is 20 cm.

„ „ a side is $(20+10=)$ 30 cm.

$\therefore$ The force on the base $= (400 \times 40 \times 1 =)$ 16000 gm. wt.

„ „ top $= (390 \times 20 \times 1 =)$ 7800 gm. wt.

„ „ a side $= (400 \times 30 \times 1 =)$ 12000 gm. wt.

Note :— Find a satisfactory explanation for the force on the base which exceeds the weight of water in the vessel.

39. Find the total force against a lock-gate 10 m. wide, the depth of water being 20 m. and the height of the lock-gate 22 m.

Hint : The force on the lock-gate is independent of the height of the gate. It depends upon the depth of water.

40. A cubical vessel has a lid 20 cm. square. A vertical pipe leading from a side of the vessel is filled with water to a height of 5 dm. above the lid. What weight is required to be put on the lid to prevent the escape of water ?

The area of the lid of the vessel $= (20 \times 20 =)$ 400 sq. cm.

The pressure on the common horizontal surface in the pipe $= (1 \times 50 =)$ 50 gm. wt./sq. cm. Hence the pressure of 50 gm. wt./sq.cm. is to be balanced by the weight W to be put on the lid. But the area of the lid is 400 sq.cm.

$$\therefore \frac{W}{400} = 50 \text{ and } W = 20 \text{ kg.}$$

41. A boy stands on a hydrostatic bellows and finds that he is supported when water is 20 in. high in the tube. If the top of the bellows is 12 in. square, find the weight of the boy. The water in the bellows weighs 63 lb. per cu. ft.

3. U-tubes

42. A column of 20 cm. of a specimen of oil in one arm of a U-tube is balanced by a column of water 17 cm. in the other arm. Find the density of the specimen.

If d is the density of oil then the pressure exerted by the column $= (1 \times 20 \times d =)$ $20d$ gm. wt. The pressure exerted by the column of water of the common horizontal surface $= (1 \times 17 \times 1 =)$ 17 gm.wt. As the columns balance each other, we have $20\,d = 17$.

$$\therefore \quad d = 0{\cdot}85 \text{ gm./cc.}$$

43. The lower portion of a U-tube contains mercury. A liquid of unknown density is poured in one of the arms, till the mercury levels in the two differ by 2 cm. Find the density of the liquid, if the column measures 30 cm.

44. The diameters of the arms of a U-tube containing mercury are 1·10 and 1·45 cm., their respective lengths being 23 and 27 cm. A column of 10 cm. of turpentine in one arm is balanced by a column of 8·7 cm. of water in the other. Find the sp. gr. of turpentine.

Hint: The pressures acting at the common horizontal surface at the bases of the liquid column are independent of the area of the sections or the total lengths of the arms of the U-tube.

45. Two tubes have their upper ends connected by a Y-piece and their lower ends are dipping, one in a salt solution and the other in water. The air is partly exhausted from the Y-piece and the solution is found to rise by

26·9 cm. and the water by 28·7 cm. in the respective tubes. Find the specific gravity of the solution.

46. One of the limbs of Hare's apparatus contains a column of a liquid (sp.gr. 1·2) 20·1 cm. high; while the other contain a column of oil 26·8 cm. high. What is the specific gravity of the oil.?

47. A mercury barometer reads 30 in. while a glycerine one shows a reading of 325 in. Find the density of glycerine.

4. Hydraulic Press

The force F exerted by the larger piston of a hydraulic press is to the force f acting on the smaller piston as the area A of the cross-section of the larger piston is to that a of the smaller piston, or

$$\frac{F}{f} = \frac{A}{a} = \frac{D^2}{d^2}$$

where D, d are the diameters of the pistons respectively.

48. The area of the smaller piston of a hydraulic press is 6 sq. in., while that of the larger one is 60 sq. in. Find the load that can be raised by a force of 30 lb.wt. applied to the smaller piston.

49. A load of 10000 kg.wt. can be raised by a Bramah's press, when a force of 100 kg.wt. is applied to the smaller piston. Find the ratio of the areas of the two pistons.

50. The pressure in the city main is 35 lb.wt./in^2. The diameter of the plunger of a hydraulic elevator is 12 in. Find the load which the elevator can lift, (1) assuming that there is no friction, (2) if the loss due to the friction is 25%.

Hint: Here the pressure that is exerted on the plunger is given. Using the known value of its diameter, determine the load that can be raised.

51. If the diameter of the cross-section of the smaller piston of a hydraulic press is 8 cm. and that of the larger piston 32 cm., (1) through what height will the larger piston rise when the smaller piston descends through 16 cm. ? (2) What force should be applied to the smaller piston to raise a load of 8 tons ?

Hint : The volume covered by the movement of the smaller piston must be equal to that due to the movement of the larger one.

52. If the radius of the cross-section of the smaller piston of a hydraulic press is 4 cm. and that of the larger piston 40 cm., and the force that is applied to the end of the lever 35 cm. long is 16 lb.wt. find the load raised by the larger piston. The lever is attached to the smaller piston at a distance of 5 cm. from the fulcrum.

The mechanical advantage of the lever is ($\frac{35}{5}=$) 7. The force exerted on the smaller piston is, therefore, ($16\times 7=$) 112 lb.wt. when 16 lb. force is applied to the end of the lever. The area of the larger piston is 100 times that of the smaller one, hence the total load that can be raised by the larger piston is ($100\times 112=$) 11200 lb.wt. *or* 5 tons wt.

53. The radii of the cylinders of a hydraulic press are 10 in. and 6 ft. respectively. The power is applied at the end of a lever 3 ft. long and the piston is attached at a distance of 4 in. from the fulcrum. If a body weighing 6 tons be placed upon the larger piston, find the force that must be applied to the lever. If the materials of the press will only bear a pressure of 100 lb.wt. per sq. in., find the greatest weight that can be lifted.

54. A hydraulic press is to exert a pressure of 20 tons. The ram is 10 in. in diameter and the plunger of the

pump supplying pressure is 2 in, in diameter. Find the pressure to be applied to the pump plunger (1) when no energy is wasted and (2) when the efficiency of the machine is 80 per cent, At each stroke the small piston moves over 12 in. Find the number of strokes required to raise the weight by 24 ft.

5. Archimedes' Principle

"When a solid is immersed wholly or partly in a fluid (*i.e.* liquid or gas) it displaces a volume of the fluid equal to the volume of the immersed part, and it experience an upward thrust, due to the fluid, equal in magnitude to the weight of the volume of the fluid displaced."

PRINCIPLE OF ARCHIMEDES

When a body of mass m and volume v is suspended by a string in a liquid of density d, then its apparent weight *i.e.* the tension in the string $= (m - vd)\,g.$,

Apparent wt. of a body in a fluid

= Actual wt. of the body − upthrust of the fluid on it.

$$\because \text{Sp.gr. of a substance} = \frac{{}^{*}\text{wt. of substance in air}}{\text{wt. of equal vol of water}}$$

$$\therefore \text{Sp.gr. of a solid} = \frac{\text{wt. of solid in air}}{\text{wt. of water displaced}}$$

$$i.e. \quad ,, \quad ,, \quad = \frac{\text{wt. of solid in air}}{\text{loss of wt. of solid in water}}$$

$$\text{and sp.gr. of a liquid} = \frac{\text{loss of wt. of solid in the liquid}}{\text{loss of wt. of sold in water}}$$

55. What is the buoyant force on 50 cc. of lead under water?

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* To be more precise, the solid should be weighed *in vacuum* and not *in air*. However, for ordinary purposes, the weights are taken in air, as the loss of weight in air is negligible.

Hint : Find the weight of the displaced volume of water. The buoyant force is due to this weight of water.

56. A piece of cork (sp. gr. 0·25) occupies a volume of 4 cc. It is kept below the water surface by means of a string fastened to the bottom of the vessel containing water. Find the tension in the string.

Hint : The tension in the string is equal to the weight of the piece *in* the liquid, *or* the apparent wt. of the solid.

57. Determine the loss of weight which an iron piece 10 cc. in volume would suffer when weighed (1) in water, (2) in oil of sp.gr. 0·8.

57*a*. A solid weighs 30 gm. in air, 26 gm. in water, and 25·2 gm. in some other liquid. Find the sp.gr. of the liquid. Determine also the volume of the solid.

Loss of wt. of the solid in water $= (30 - 26 =)$ 4·0 gm. wt.

„ „ „ in the liquid $= (30 - 25{\cdot}2 =)$ 4·8 gm.wt.

$\therefore$ sp.gr. of the liquid $= (4{\cdot}8/4{\cdot}0 =)$ 1·2.

Again the vol. of water displaced by the solid $= (\frac{4}{1} =)$ 4 cc.

$\therefore$ the vol. of the solid $=$ 4 cc.

58. A piece of glass, weighing 30 gm. in air, weighs 18 gm. when suspended in water and weighs 19·56 gm. when immersed in turpentine. Find the sp. gr. of the glass piece and of turpentine.

59. A solid, weighing 2·65 gm, in air, is attached to a sinker which alone weighs 8·80 gm, in water. Both together are found to weigh 8·55 gm. in water. What is the sp. gr. of the solid ?

Wt. of the solid in water $= (8{\cdot}55 - 8{\cdot}80 =) - 0{\cdot}25$ gm.

(We get *negative* sign because upthrust is greater than the weight of the solid acting downwards.)

Loss of wt. of the solid in water $= [\,2{\cdot}65 - (-0{\cdot}25) =\,]$ 2·90 gm. wt. $\therefore$ sp.gr. of the solid $= \dfrac{2{\cdot}64}{2{\cdot}90} = 0{\cdot}91$.

60. A heavy solid weighed 16 gm. in air and 14·1 gm. in water. When tied to a piece of paraffin wax, weighing 5·03 gm. in air both together weighed 13·42 gm. in water. Determine the specific-gravities of the solids.

61. A piece of lead weighs 5·6 gm. in air, 5·1 gm. in water, 5·185 gm. in alcohol and 5·08 gm. in a salt-solution. A piece of wood weighs 1·6 gm. in air and when tied to the lead piece both together weigh 4·7 gm. in water. Determine the sp. gr. of lead, wood, alcohol and of the salt-solution.

62. A piece of ebonite (sp. gr. 1·8) weighs 18 gm. in air, 8·9 gm. in linseed oil and 5·4 gm. in glycerine. Determine the sp. gr. of the oil and of glycerine.

Hint: Sp. gr. of ebonite piece is given. That gives the weight of equal volume of water. Hence calculate the sp. gr. of the liquids.

63. A crystal of aluminium sulphate* weighs 16·22 gm. in air and 9·12 gm. in petrol of sp. gr. 0·7. Find the density of the crystal.

$$\because \frac{\text{Loss of wt. of the solid in petrol}}{\text{Loss of wt. of it in water}} = 0{\cdot}7.$$

the loss of wt. of the solid in water $= (16{\cdot}22 - 9{\cdot}12)/0{\cdot}7$

$= 7{\cdot}1/0{\cdot}7$ gm.

$\therefore$ the sp. gr, of the crystal $= (16{\cdot}22 \times 0{\cdot}7/7{\cdot}1 =) 1{\cdot}60$

But the density $=$ sp. gr. $\times$ density of water

$\therefore$,, $= 1{\cdot}60 \times 1 = 1{\cdot}60$ gm./cc.

64. A crystal of potassium bichromate weighs 1·284 gm. in air, and 0·857 gm. in rock-oil (sp. gr. 0·82). Determine the sp. gr. of the crystal.

* This method is used to find the sp. gr. and density of a solid soluble in water. Any liquid in which the solid does not dissolve, can be used. Note that greater care is required, if the liquid chosen is *inflammable*.

65. A solid weighs 10 gm. in air of density 1·293 gm. per litre and 6·8 gm. in alcohol (sp. gr. 0·8). Find the density of she solid.

The wt. of the solid in vacuum = its wt. in air + the wt. of air displaced ; and the wt. of it in alcohol = its wt. in vacuum – the wt. of alcohol displaced.

Hence assuming v as its volume and d the density :

$$vd = 10 + 0{\cdot}001293\ v;\ \text{and } 6{\cdot}8 = vd - 0{\cdot}8\ v.$$

$$\therefore\ v = 4{\cdot}006 \text{ cc. and } d = 2{\cdot}498 \text{ gm /cc.}$$

66. A lump of sugar 40 gm. in weight is coated with wax and weighed in air. The weight is 40·92 gm. Then it is weighed when immersed in water. The weight is found to be 14·92 gm. Determine the density of the lump. (Density of wax = 0·92 gm./cc.)

The volume of the wax-coating is

$$\frac{\text{mass}}{\text{density}} = \frac{40{\cdot}92 - 40}{0{\cdot}92} = 1 \text{ cm.}$$

The weight of water displaced by the coated lump is (40·92 – 14·92 =) 26 gm.wt. therefore the volume of the coated lump is 26 cm. Hence the volume of the un-coated lump of sugar is (26 – 1 =) 25 cc.

$$\therefore\ \text{the density} = \frac{\text{mass}}{\text{volume}} = \frac{40}{25} = 1{\cdot}6 \text{ gm./cc.}$$

67. A piece of cork (20 cc.) is attached to a piece of brass (2 cc.) and weighed when completely immersed in water. If they are found to have no weight, determine the sp.gr. of the cork. (The density of brass is 8·4 gm./cc.)

Hint : The downward force due to the wt. of the solids just equals the upthrust of the liquid. Volume of the solids are known. Hence find the weight of the cork in air. Then the sp. gr. Mass of the brass piece be found from its density and volume.

*68. A brass sphere (6 cm. in diameter) is being weighed in water. It just balances at the other end of the beam an aluminium cylinder of the same radius. The aluminium piece is also kept immersed in water. Find the length of the cylinder. (Sp. gr. of brass is 8·4 and of aluminium 2·7)

69. What force would be required to support a cubic centimetre of platinum in mercury? The density of platinum is 21·5 gm./cc. and the sp. gr. of mercury 13·6

70. The weight of a submarine boat is 400 tons. It lies damaged and full of water at sea. What pull should be exerted by the lifting chains in order to lift it from the bottom? The sp. gr. of submarine material is 7·8 and of sea water 1·025, the density of pure water being 62·5 lb./cu. ft.

Hint: The chains have to overcome the downward force equal to the weight of the submarine material *in the water.*

71. A sphere of radius 10 cm. is just wholly immersed in a liquid of sp. gr. 1·6. Find the resultant vertical pressure on it due to the liquid. Will the value be the same if the depth of the sphere under the liquid be varied?

The resultant vertical pressure is the upthrust of the liquid equal to the weight of the liquid displaced $=(\frac{4}{3}\pi \times 10^3 \times 1{\cdot}6=)6703$ gm. wt.

The resultant vertical pressure remains the same whatever be the depth of the sphere.

*72. Two solid spheres each of 10 cm. radius are connected by a light string and totally immersed in a vessel of rockoil (sp.gr. 0·81). If the sp.gr. of the spheres be 0·3 and 2·5 respectively, find the tension in the string and the pressure between the heavier sphere and the bottom of the vessel.

Hint: The total weight of the spheres minus the weight of the liquid displaced by them is the required pressure. The tension is equal to the weight of the lighter sphere *in the liquid.*

73. Determine tbe acceleration with which a piece of brass (sp. gr. 9·0) would just sink in water and the time which it would take to get to the bottom of a pool 14·22 ft. deep. Assume the acceleration to remain unchanged.

Hint : The *weight* of the brass piece *in water* acts as the moving force on its mass. Find the acceleration and hence time.

***74.** The weight of an empty flask with stopper is 20 gm. The density of glass is 2·5 gm./cc. The flask when full of salt-solution weighs 49·50 gm. While still full of the solution, it is suspended by a thread in water and again weighed. The weight is 16·50 gm. Find the density of the solution.

> The volume of the material of the flask is (20/2·5 =) 8·0cc. The weight of water displaced by the flask is (49·50–16·50 =)33 gm. wt. Hence the volume of the flask is 33 cc., therefore its capacity or the volume of the solution in the flask is (33 – 8 =) 25 cc. But the mass of the solution is (49·50 – 20 =) 29·50 gm. therefore its density is (29·5/25 ±) 1·18 gm./cc.

***75.** A hollow stopper of glass (sp. gr. 2·6) weighs 27·3 gm. in air and 5·75 gm. in water. What is the volume of the internal cavity ?

Hint : First find the total volume of the stopper from the loss of wt. in water. From its mass in air and the sp. gr., of the glass, determine the volume of the material of the stopper. Hence calculate the capacity of the cavity.

***76.** A hollow glass bulb (sp. gr. 2·3) is found to weigh

34·5 gm. in air, and 6·28 gm. in a liquid of sp.gr. 0·83. Find the capacity of the bulb.

77. A diamond ring weighs 12 gm in air and 11·16 gm. in water. Find the mass of the diamond if the sp. gr. of gold used is 17·5 and that of diamond in it 3·5.

*78. A vessel completely filled with water is placed on a spring balance (in the pan at the top) and the total weight indicated is 6 lbs. A piece of iron, weighing 4 lbs. is suspended from another spring balance so that it is completely immersed under the water. The water spilt out is collected in the pan. The upper balance now shows 3·5 lbs. wt., while the lower one registers 6·5 lbs. wt. The quantity of water collected in the pan is found to weigh 0·5 lb. wt.

What do you deduce from the above experiment? How do you account for the increase of weight to 6·5 lbs. wt.? Find the density of the solid.

6. Principle of Floatation

When a body floats in a liquid partly submerged in it, the weight of the liquid displaced equals the weight of the body. Hence specific gravity of a light solid (*i.e.* a solid which floats in water $= \dfrac{\text{volume of the solid submerged in water.}}{\text{total volume of the solid}}$

79. A piece of ice having a volume of 24·5 cu. in. floats in water with 22·05 cu. in. of it under the surface. Determine its specific gravity.

80. A rectangular block of wood 10 in. high sinks 7 in. in water and 8 in. in a specimen of oil. Find the sp. gr. of the wood and of the oil.

81. A man 71·4 kg. in weight just floats in sea-water (sp.gr. 1·02). What is his volume?

Hint: The weight of the displaced water just equals the weight of the man. Find the volume of the displaced water.

82. A ferry barge with vertical sides has a bottom $20' \times 30'$. When some motor cars were on board, it floated 4 in. deeper in water than before. Determine the weight of the cars.

83. A steamer weighs 1200 tons. Find its displacement in sea water of density 64 lb./cu. ft. On loading it with a cargo, it was found to sink in water by 4 in. more. Find the weight of the cargo, if the mean sectional area of the steamer is 3000 sq. ft.

84. A rod floats with $\frac{3}{4}$ of its volume submerged in a liquid of sp.gr. 1·2 ?

85. A diver with his diving suit weighs 100 kg.. It requires 14·125 kg. of a lead piece, under water, *just* to sink him. If the density of lead is 11·3 gm./cc., what is the volume of the diver and his suit.

The wt. of the displaced water is equal to the wt. of the diver with his suit *plus* the wt. of the lead piece. Hence, if v cc. be the volume of the diver and his suit, we have

$$\left(v + \frac{14125}{11 \cdot 3}\right) \times 1 = 100000 + 14125 \quad \therefore \; v = 112875 \text{ cc.}$$

86. A man, weighing 128 lb., can float just in sea water. He is able to keep 0·5 cu. ft. of his body above the surface of the water by holding a piece of wood (sp. gr. 0·68) under water. Find the sp. gr. of sea water, if the volume of the wooden piece is 1·5 cu. ft. and the density of pure water 62·5 lb./cu. ft.

Hint: Weight of the additional volume of water displaced equals the weight of the wooden piece.

87. A wooden cube just floats in water when supported by a force of 98 gm. wt. On removing this extra force the cube rises by 2 cm. above the surface. What is the size of the cube ?

Hint : The resultant upthrust of water on the cube is balanced by the extra force of 98 gm. wt. This gives the wt. of the water displaced by 2 cm. length of the cube. Find the cross-section of the cube, thence the length of a side.

88. Equal volumes of gold and iron (sp.gr. 7·82) are tied together and immersed in mercury (sp.gr. 13·6). They are found to float just in the liquid. Find the sp. gr. of gold.

Let v cc. be the volume of each piece and d the density of gold, then $v \times 7{\cdot}82 \times 1 + v \times d = 13{\cdot}6 \times 1 \times 2v$.

$\therefore \quad d = (27{\cdot}2 - 7{\cdot}82 =)$ 19·38 gm/cc.

Hydrometers

89. A cylindrical test-tube with a uniform cross-section contained some lead shots at the bottom so as to make it float vertically. When put in water, it floated with 5·25 cm. of its lenth below the surface, and when put in a salt-solution 5·1 cm. of it were in the liquid. Find the density of the solution.

Let A be the area in sq.cm. of the cross-section of the tube, then the volume of water displaced $= (5{\cdot}25 \times A)$cc. and the mass of the displaced water

$= (5{\cdot}25 \times A \times 1)$ gm. $\qquad (M = V \times D)$

similarly the mass of the displaced solution

$= (5.1 \times A \times D)$ gm., where D is the density of the solution. But these masses are each equal to the mass of the tube. $\quad \therefore \quad 5{\cdot}1 \times A \times D = 5{\cdot}25 \times A \times 1$

$\therefore \quad D = 1{\cdot}03$ gm./cc.

90. A rectangular brass tube is loaded to float vertically in water. It floates in water with 12 in. of it inside, and in a liquid of unknown specific gravity with 14 in. of it submerged. Determine the specific gravity of the liquid.

91. A pencil has a lead piece attached to one end. It floates in water with 2 cm. projecting above the surface and in a liquid with 3·3 cm. projecting. The total volume of the metal and the pencil is 5 cc. and the cross-section of the latter 45 sq mm. Determine the sp.gr. of the liquid.

Hint: Find the volumes of the liquids displaced by the pencil. Hence equate the masses of the liquids displaced by it.

92. A common hydrometer of volume 20 cc. is floated in water. The ratio of the volume of the hydrometer remaining in water to that of the stem outside water is 3 : 1. The cross-section of the stem is 1 sq. cm. Find the length of the stem by which the hydrometer will go down in a liquid of density 0·8 gm./cc. when it is placed in the liquid. [*BA*.1928]

***93.** A variable immersion hydrometer floates in liquids *A*, *B* and *C* with 3 in., 3·5 in. and 4·5 in. of its stem outside the liquids respectively. If the sp.gr. of *A* and *B* are respectively 0·75 and 0·80, find that of *C*.[*S.E*.1922]

Hint: Assume v to be the volume of the hydrometer and a the cross-section of the stem, then find the volumes immersed in different liquids. Equating the weights of *A* and *B*, find v in terms of a, and hence equating the weight of the displaced volume for *C* with that for *A* (or *B*) determine the required sp.gr.

94. A brass tube of uniform cross-section weighs 40·2 gm. when empty. A weight of 31·8 gm. is required to be added to it, in order to make it sink in water upto

a certain mark on it. When the same tube is made to float in some other liquid, an additional weight of 18·72 gm. is required to make it sink to the same mark. Find the specific gravity of the liquid.

Hint: Equal volumes of the liquids are displaced. Find their weights. Hence the specific gravity of the liquid.

95. An empty hydrometer weighing 500 gm. floates in water immersed up to a particular mark on the stem. When allowed to float in a salt solution 88 gm. have to be added to the upper pan to make it sink to the same mark. Determine the sp.gr. of the liquid.

Hint: The mass of the water displaced by the hydrometer = 500 gm. (which is the mass of the hydrometer); while, the mass of the same volume of the displaced liquid is (500+88) gm.

96. The following observations were taken with a Nicholson's hydrometer.

Mass required to sink the hydrometer to the mark = 8·278 gm.
Mass required to sink the hydrometer to the mark when the solid is in the upper pan = 0.102 gm.
Mass required to sink the hydrometer to the mark when the solid is in the lower pan = 3·022 gm.
Determine the sp. gr. of the solid.

Mass of the solid in air = (8·278 − 0·102 =) 8·176 gm.
Apparent mass of the solid in water = (8·278 − 3·022 =) 5·256 gm.
Apparent loss of mass of the solid = (8·176 − 5·256 =) 2·920 gm.
Mass of the equal volume of water = 2·92 gm.

$\therefore$ sp.g. of the solid $= \frac{8 \cdot 176}{2 \cdot 92} = 2 \cdot 8$.

97. To sink a Nicholson's hydrometer to the mark in water, 4 gm. are required. When a piece of wax is put in the upper pan, only 3 gm. are required to sink it to the same mark. On putting the same piece in the lower pan 4·1 gm. are required to make it sink to the mark. Find the density of the wax piece.

Hint : Note the proper algebraic signs for the mass in water and the apparent loss of mass of the solid.

Gas-Pressure

98. Find the atmospheric pressure per sq.cm. corresponding to a pressure of 76 cm. of mercury.

The mass of the given mercury column of unit cross-section $= (76 \times 1 \times 13 \cdot 6)$ gm. $\therefore$ the required pressure $= (76 \times 1 \times 13 \cdot 6 =)$ 1033·6 gm. wt./cm.^{2} *or* 1012928 dynes-cm^{-2}.

99. Find the average height of the atmosphere above the earth's surface, taking the density of air as 1·29 gm. per litre and the atmospheric pressure equal to 76 cm. of mercury.

100. A U-tube manometer is filled with some mercury at the bent. A man blows into one of the arms of the tube and thereby creates a difference of 4 cm. in the levels of mercury in the two arms. What pressure (in dynes) is he exerting?

101. Find the pressure exerted by the air confined in a faulty barometer registering 74·23 cm., if a correct barometer registers 75·64 cm.

Hint : The pressure due to the air enclosed *plus* the pressure of the mercury column in the faulty apparatus must equal the atmospheric pressure, which in turn is

equal to the pressure of the mercury column in the correct barometer.

102. Taking the pressure of the atmosphere equal to a column of 32 ft. of water, determine the pressure of air in the barrel of a pump, when the water stands in the tube 20 ft. above the level of the reservoir.

103. A U-pressure gauge attached to a boiler chimney reads 1·36 in. of water. If the barometer reads 30·1 in. of mercury, find the absolute pressure inside the chimney. (Density of water = 0·03611 lb./cu. in.)

> 1·36 in. of water are equivalent to 0·1 in. of mercury Hence the required pressure = (30·1 − 0·1 =) 30 in. of mercury *or* ($30 \times 13{\cdot}6 \times 0{\cdot}03611 \times 32 =$) 471·3 plds. in.$^{-2}$

Note :— The pressure in a boiler chimney is always less than the pressure of the atmosphere outside,

104. The mechanical advantage of the arm of a safety valve is 5 and the valve has got 2 cm. diameter. Find the steam pressure, if 44 lb. wt. is required to be applied to the end of the arm.

> The downward force F exerted at the safety valve is equal to the product of the mechanical advantage and the power applied to the arm, therefore $F = (5 \times 44 =)$ 220 lb. wt. but this force is equal and opposite to the upward thrust of the steam which acts on the valve of area ($\pi \times 1^2 =$) π sq.cm. Hence the steam pressure is [$220/(\pi \times 1^2) =$] 70 lb,wt./sq cm.

105. A safety-valve is kept in position by a horizontal lever 30 cm. long having its fulcrum at one end and a weight of 10 lb. on the other. The area of the valve is 6 sq. in. and its centre is 3 cm. from the fulcrum. Find the greatest pressure the steam in the boiler can exert.

106. Find the total lifting power of a balloon, the bag of which weighs 120 gm. and is filled with 30 cu.m. of hydrogen gas. The density of air is fourteen times that of hydrogen.

Suppose d is the density of hydrogen in gm.per cu.m.
then $14d$,, ,, air ,, ,, ,,
Hence the upthrust due
to the displaced air $= (\ 14d \times 50\ )$ gm. wt.
But the downward force $= (\ 120 + 50d\ )$ gm. wt.
$\therefore$ the resultant upthrust $= [700d - (120 + 50d)]$gm.wt.
$\therefore$ the lifting power $= (650d - 120)$ gm.wt.

107. A balloon is filled with a gas and weighs 62·5 kg. The weight of the empty balloon is 2·5 kg. The density of the gas is $\frac{1}{7}$ times that of air. Find the force with which the balloon will rise up, if the density of air is 1·29 kg. per cu. m.

108. A litre of air weighs 1·29 gm. and a litre of coal gas weighs 0·79 gm. Find the normal capacity of a balloon which will just ascend. The weight of the balloon including the cargo is 1000 kg.

Hint: The lifting power is (1·29 – 0·79 =) 0·50 gm. per litre. The total lifting power is expected to be 1000 kg. Hence determine the total volume of the balloon.

109. An airship 200 metres long has an average diameter of 20 metres and is charged with hydrogen. If the balloon and its attachments weigh 20000 kgs., find the further weight which may be carried. (Density of hydrogen 0·09 gm./litre. Density of air 12.9 gm./litre.)[I.Sc. 1935]

9. Boyle's Law—applications

The volume of a given mass of a gas varies inversely as its pressure, the temperature remaining constant.

Hence, the density of a given mass of a gas varies directly as the pressure, provided the] temperature is constant.

$$\therefore \quad P_1 V_1 = P_2 V_2, \frac{P_1}{d_1} = \frac{P_2}{d_2}, \text{ temp, being constant.}$$

110. What volume will a litre of air occupy, if the pressure on it is changed from 751·5 to 600 mm, of mercury, the temperature remaining constant ?

From Boyle's law we have, $P_2 V_2 = V_1 P_1$, $\therefore$ substituting the known values in the formula, we get,

$V_2 \times 600 = 1 \times 751{\cdot}5 \quad \therefore \quad V_2 = 1{\cdot}2525$ litres.

111. What is the pressure inside a motor tyre if its internal volume is 5 cu.ft. and 9 cu.ft. of air at atmospheric pressure are used in pumping it up ? [I.Sc. 1935]

112. The inner tube of a motor tyre may be taken as a cylinder of 100 mm. diameter and 1000 mm. long. What volume of air measured at atmospheric pressure must be pumped in to raise the pressure to 5 atmospheres ? [B. A. 1924]

113. A glass globe will just stand an excess of 2 lb. per sq. in. of inside pressure over that on the outside. The bulb is sealed up at a place, where the atmospheric pressure is 14·5 lb./sq. in. and then taken uphill until it bursts. Find the atmospheric pressure at the place, where the globe bursts.

Hint : The inside pressure remains the same, while the outside one fails till the two differ by 2 lb./sq. in. Hence find the low pressure uphill.

114. A barometer tube contains a little air in the space above mercury. This space is 12·5 cc., when the mercury level in the tube is 67 cm. above that in the cistern. On raising the tube, the volume of air in the

tube changes to 20 cc. and at the same time the mercury column increases to 70 cm. Find the true barometric height.

Let H cm. be the true barometric height then $(H-67)$ cm. is the pressure exerted by the gas when the volume is 12·5 cc., and $(H-70)$ cm. is the pressure, when it occupies 20 cc. Hence, applying Boyle's Law, we get,

$(H-67)\,12\text{-}5=(H-70)\,20 \quad \therefore \; H=75$ cm. of mercury.

115. An ordinary barometer having a uniform bore has some air enclosed in the space above mercury. The height of the mercury column in the tube is 29 in. and the space above it is 4 in. long. The tube is then pushed down into a cistern with the result that the mercury column becomes 28 in. long and that space above it only 2 in. long. If the barometer had been perfect, what would have been the height of the barometer column? [B.A. 1926]

116. When the atmospheric pressure is 75 cm. of mercury, air is introduced into a barometer tube of length 80·5 cm. The level of mercury, thereby, falls to 68 cm. On the next day, the height is found to change to 67 cm. temperature remaining the same. What is the atmospheric pressure on that day?

117. A capillary tube of uniform bore, closed at one end has some air enclosed in it by a thread of mercury 20 cm. long. When the tube is kept horizontal on a table, the air column measures 45 cm. and when held vertical with the open end upwards, the column of air measures 35 cm. Determine the atmospheric pressure.

Hint: When the tube is vertical, the pressure exerted by the enclosed air equals the atmospheric pressure *plus* that due to the mercury column in the tube. When

horizontal, the pressure exerted equals the atmospheric pressure only.

118. A bubble of air occupies a volume of 3 cu. in. at a depth of 13·6 ft. from the surface of water in a lake. Find the depth at which it would occupy 2 cu. in., if the atmospheric pressure is 30 in. of mercury.

Since the density of mercury is 13·6 times that of water, the pressure due to a column of 30 in. of mercury equals the pressure due to a column of water 34 ft. long.

Hence $P_1 = (\ 13{\cdot}6 + 34 = \ )\ 47{\cdot}6$ ft. water

and $P_2 = ?$ (the quantity to be determined).

$V_1 = 3$ cu.in. and $V_2 = 2$ cu.in.

$\therefore$ from Boyle's law, we get, $P_2 = \dfrac{47{\cdot}6 \times 3}{2} = 71{\cdot}4$ ft. of water. Hence the depth of the bubble below the surface $= (\ 71{\cdot}4 - 34 = \ )\ 37{\cdot}4$ ft.

119. A uniform tube 79·8 cm. long and 1 sq. cm. in cross-section is filled with mercury and inverted with the open end one cm. below mercury so as to form a barometer. The height of the column is found to be 75 cm. The true atmospheric pressure is 76 cm. If the error of the barometer is due to the entrapped air, what volume of air was so entrapped ? [I.Sc. 1926]

The enclosed air occupies a volume of [79·8 – (1+75) =] 3.8 cc. and exerts a pressure of (76 – 75 =) 1 cm. If v be the volume of the same air outside at 76 cm. pressure, then from Boyle's law,

$76 \times v = 1 \times 3{\cdot}8, \qquad \therefore \quad v \times 0{\cdot}05$ cc.

120. The volume of the tube of a bicyle tyre is 100 cu.in. and the barrel of the pump 10 cu.in. Find the number of strokes required to produce double the

atmospheric pressure in the tube, assuming its volume to increase by $\frac{1}{10}$ when it is inflated. [I.Sc. 1928 Madras]

Hint: Using Boyle's law, find the total volume v_1 of air at atmospheric pressure which will have to be pumped into the tube having a volume 100 cu. in. when inflated. Every stroke of the piston forces a volume of 10 cu. in. of air. Find the number of strokes. It already contains 100 cu. in, of air.

121. A glass sounding-tube (tube used to determine the depth of a sea) is 40 cm. long and is open at the lower end, the inside having been coated with a soluble pigment. It is lowered to the bottom of a sea. On taking it out at the surface, the pigment is found to have been washed off upto 30 cm. height in the tub. Find the depth of the sea, if the reading on a water barometer is 9 m. and the sp. gr. of sea-water 1·02.

Hint: First find the total pressure which the enclosed air is subjected to. From that find the depth of the sea-water below the surface upto the level of the water in the tube, Hence the depth of the sea is equal to this depth *plus* the height of the water column in the tube.

***122.** A diving bell 10 ft. high carries a mercury barometer in its inside. Before it was lowered in water, the barometer reads 30 in. After reaching a certain depth it was fonud to read 35 in. Find the depth of the top of the bell and the height to which water rises in the bell.

Find the volume of air at the atmospheric pressure which should be pumped in, in order to fill the bell at this depth. [I.Sc. 1915]

The pressure increases by ($35 - 30 =$) 5 in. due to the column of water ($5 \times 13{\cdot}5 =$) 68 in. in height above the inside level of water. Now applying

Boyle's Law $10 \times 30 = X \times 35$ where X is the height of air column inside, we get $X \times \frac{60}{7}$ ft. of air column. Hence the water rises in the bell by $(10 - \frac{60}{7} =)\ 1\frac{3}{7}$ ft. while the top of the bell is outside the surface of water by

$$10 - \tfrac{17}{3} - \tfrac{10}{7} = 2\tfrac{19}{21} \text{ ft.}$$

Keeping the inside pressure at 35 in. the air outside originally at 30 in. has to be pumped in so as to displace the water column of $1\frac{3}{4}$ ft. Hence if v is unknown volume of air, $v \times 30 = 1\frac{3}{7} \times 35 \times A$. $\therefore\ v = 1\frac{2}{3}\ A$ cu. ft. of air where A is the cross-section of the bell.

123. The density of air at 760 mm. of mercury, is 1·293 gm./litre. Find its value, if the pressure is reduced to 680 mm. of mercury. the temperature remaining the same.

124. A gas cylinder of capacity 5 cu.ft. contains air at a gauge pressure of 210 lb. sq.in. Find the volume this gas would occupy at an outside pressure of 14 lb./sq. in.

Since a pressure gauge indicates a pressure above the atmospheric pressure, the pressure of the air in the

cylinder $= (210+14=)\ 224$ lb./in.^{2}

while the volume occupied $= 5$ cu.ft. Hence if this air were at a pressure of 14 lb./in.^{2} the volume V_2 it would occupy is given by $\dfrac{P_1 V_1}{P_2} = \dfrac{224 \times 5}{14}$. $\therefore\ V_3 = 80$ cu. ft.

125. Determine the density of air within an automobile tyre that is inflated to a gauge pressure of 21·3 lb./in.^{2} when the atmospheric pressure is 14·2 lb./in.^{2}.

10. Air pumps

In an air pump if V and v are the volumes of the receiver and of the barrel respectively the density of the

remaining air after n complete strokes is

$$d_n = \left(\frac{V}{V+v}\right)^n \times \text{original density of air} . \text{(i)}$$

and the final pressure

$$P_n = \left(\frac{V}{V+v}\right)^n \times \text{original pressure in it. (ii)}$$

126. The volume of the receiver of an air-pump is five times that of the barrel. Find the number of complete strokes required to have the density of the remaining air $\frac{125}{216}$ of the original density.

If n be the number of strokes required, then using the general relation with the usual symbols we have

$$d_n = \left(\frac{V}{V \times v}\right)^n \times d \quad \therefore \left(\frac{5}{6}\right)^n = \left(\frac{125}{216}\right) = \left(\frac{5}{6}\right)^n \quad \therefore n = 3.$$

127. Find the ratio of the volume of the receiver to that of the barrel of an air pump in which the density of the air remaining after three complete strokes is to that of the original air as 216 to 343.

128. What would be the reading of a barometer under the receiver of an air-pump after 4 complete strokes if the volume of the barrel is one-fifth the volume of the receiver and the atmospheric pressure 30 in.

Hint: As the air is being withdrawn, the pressure of the remaining air inside the receiver decreases, hence the barometer reading falls. Find the pressure after four strokes.

129. Under the receiver of an air-pump is placed a balance carrying two spheres one in each pan. One of these spheres is 3 cm. in diameter and weighs 40·032 gm. and the other is 5 cm. in diameter and weighs 40·066 gm. in vacuum. On getting a partial vacuum, these spheres are found to balance each other. What is the reduced pressure

in the receiver? Density of air at $N.T.P.$ is 1·293 gm /litre.

Hint: Apply the principle of Archimedes. The apparent weights of the spheres become equal on reducing the density of the air in the receiver. Determine this density by equating the apparent weights and hence the pressure.

130. A common pump is used to raise water from a well. Find the greatest height at which the lower valve of the pump can be placed above the surface of water in the well, when the barometer reads (1) 27 in., (2) 30 in. of mercury.

Hint: The height must not exceed the height of the water-barometer.

***131.** The length of the lower pipe of a common pump above the surface of water is 10 ft., and the area of the cross-section of the barrel is 4 times that of the pipe. Taking 33 ft. as the height of the water barometer, find how far the water will rise at the end of the first stroke, if the length of this be 3 ft. 6 in. [S.E.1916]

If A be the section of the pipe, the volume of air in the pipe at a pressure of 33 ft. is $10A$ cu.ft. On raising the piston, the volume of air changes to $(10A+3{\cdot}5\times 4A-xA)$ cu. ft., if x be the height to which the water rises in the pipe. The pressure of air reduces to $(33-x)$ ft. Hence by Boyle's law,

$$33\times 10A=(10A+14A-xA)(33-x) \qquad \therefore \quad x=9{\cdot}8 \text{ ft.}$$

MECHANICS

1. Velocity and Acceleration

a. Velocities

$$\text{Velocity} = \frac{\text{change of space } s}{\text{change of time } t}.$$

If a particle is imparted two velocities v_a, v_b simultaneously, it will move with a velocity v_r given by the relations $v_r^2 = v_a^2 + v_b^2 + 2v_a v_b \cos\theta$, where θ is the angle between the two velocities v_a and v_b. (If a velocity v_a of a particle is changed to v_r then the change in velocity v_b is also given by the above relation if the angle between the two velocities is known.)

Average velocity $V = \frac{1}{2}(v_o + v_t)$, where v_o is the initial velocity and v_t the final velocity of a particle.

1. An express train leaves Poona at 3–25 P.M. and reaches Victoria Terminus distant 119 mi. at 7–38 P.M. What is the average speed of the train in miles per hour ?

2. A railway train is observed to pass two points 660 ft. apart in 10 sec. Calculate its speed in miles per hour.

3. A body is moving with a velocity of 15 mi. per hour. While it is in motion, a velocity of 30 mi. per hr. is imparted to it. What is the velocity at which the body moves ?

4. A balloon starts with a vertical velocity of 400 ft. per min. and at the same time, the wind blows horizontally at a speed of 300 ft./min. to the East. Find the actual velocity with which the balloon rises up.

Hint: The balloon retains its *vertical* velocity and acquires the horizontal velocity of the wind. Find the resultant velocity with which the balloon moves. Apply the parallelogram Law.

5. The Southerly component of the wind is 10 mi./hr. and the Westerly component 15 mi./hr. Find the actual direction and the velocity of the wind.

6. An aeroplane has a speed of 60 mi. an hour in a direction due West, while N.E. wind is blowing at a speed of 20 mi. an hour. Find graphically the actual velocity with which the aeroplane moves.

Note :—Wind coming from N. E. are knwon as N. E. winds.

7. A person can row at the rate of 4 mi./hr. in still waters. He crosses a river flowing at the rate of 2 mi./hr. with the same speed as in still water. If he keeps rowing at right angles to the stream, will he land at the opposite point? If not, how far from the point will he do it? The river is 0·25 mi. broad.

8. A cricket ball is moving with a velocity of 30 mi/hr. in S.W. direction. Find the components of the velocity in the directions, due to South and due West.

9. A cricket ball, moving with a speed of 30 ft./sec., is hit in such a way, that after the blow, it moves with a speed of 40 ft./sec. and in a direction at right angles to its original direction. Find the change of velocity produced.

Let PQ represent to scale, the initial velocity of the ball and QR the final velocity. Complete the triangle PQR, then $\dot{P}R$ represents to scale the change in velocity, *or* using the parallelogram Law, we get $PR^2 = PQ^2 + QR^2$. θ being 90° $\therefore$ $PR = (\sqrt{30^2+40^2} =)$ 50 ft./sec.

***10.** A man shoots at a bird distant 100 ft. due north

of him. The bird is flying due east with a velocity of 30 ft./sec. How far ahead of the bird must he aim so as to hit it? The velocity of the shot is 500 ft./sec.

***11.** Rain appears to fall in a direction 30° to the vertical to a passenger in a running train having a speed of 30 mi./hr. What is the velocity of the rain-drops, if they are falling vertically downwards?

Hint: Suppose the velocity of the rain is vertically downwards along AM, and AC is the horizontal velocity of the train. Now make the passenger steady by imparting to the train a velocity AD equal and opposite to AC: then represent the direction in which the rain appears to fall to the passenger by AE making 30° with the vertical. Complete the parallelogram on AE as diagonal and AD as one of the sides, then AG the vertical of $ADEG$ gives the velocity of the rain-drops.

***12.** A train is travelling at the rate of 30 mi./hr. Find the direction in which a man jumping out of it at right angles to the direction of its motion with a speed of 8 ft./sec. will appear to fall.

b. EQUATIONS OF MOTION

If v_0 = Initial velocity of a particle moving in a straight line.

v_t = final velocity at the end of time t,

V = average velocity during the time t,

s = space described in time t,

α = uniform acceleration in the direction of motion.

and a particle starts from rest,

$$v_t = \alpha t \quad \ldots \quad \ldots \quad (1)$$

$$V = \tfrac{1}{2} v_t \quad \ldots \quad \ldots \quad (2)$$

$$s = \tfrac{1}{2} \alpha t^2 \quad \ldots \quad \ldots \quad (3)$$

$$v^2_t = 2 \alpha s \quad \ldots \quad \ldots \quad (4)$$

If it possesses initial velocity v_o; then, ...

final velocity $v_t = v_o + \alpha t$ ... (5)

average velocity $V = \frac{1}{2}(v_o + v_t)$ ... (6)

distance $s = v_o t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(v_o + v_t)t$ ... (7)

and $v^2_t = v_o^2 + 2as$ ... (8)

Note :—In the case of a body falling freely vertically downwards, the acceleration acting on it is equal to g. (9)

When a particle starting from rest slides down a smooth inclined plane of inclination θ, the acceleration $a = g \sin \theta$. (10)

13. A tram car, starting from rest, moves with a acceleration of 2 ft./sec². Find the velocity it would have in 10 sec. and the space described during the time.

Since $v_t = \alpha t$, here $v_t = (2 \times 10)$ 20 ft./sec.

Again $s = \frac{1}{2}\alpha t^2$ $\therefore$ $s = (\frac{1}{2} \times 2 \times 10^2 =)$ 100ft.

14. Find the acceleration generated in a car, which, starting from rest, moves over 180 ft. in 10 sec.

15. A tram car starts from rest and moves over a distance of 180 ft. with a uniform acceleration of 10 ft. per sec. Find the time required.

16. A train moving with a velocity of 30 mi./hr. on a horizontal track, is brought to rest in 20 sec. by a uniform retardation. Find this retardation.

The initial velocity of the train is 30 mi./hr. *i.e.* 44ft./sec. and the final velocity is zero. Therefore from the relation $v_t = v_o + \alpha t$, we get $0 = 44 + 20a$ $\therefore$ $a = -2{\cdot}2$ ft./sec.² $\therefore$ the retardation is 2·2 ft./sec.²

17. The velocity of a locomotive at a certain instant is 30 mi./hr. Find its velocity after 10 sec., if it moves with a uniform acceleration of 2·2 ft./sec.²

18. An automobile, running at a speed of 15 mi. per hour while passing a lamp post, gets a speed of 30 mi./hr,

while passing by the next lamp post, distant one furlong from the first. What acceleration is gained by the automobile ?

Since $\qquad v_t^2 = v_0^2 + 2as$ and $s = 660$ ft.

$v_t = 30$ mi/hr $= 44$ ft./sec., while $v_0 = 22$ ft./sec.,

we have $44^2 = 22^2 + 2 \times a \times 660. \quad \therefore\ a = 1{\cdot}1$ ft./sec.^{2}

19. A tram car passes a lamp post A at a speed of 15 mi./hr. and on passing a next lamp post B, it possesses a velocity of 30 mi./hr. Find the acceleration assuming it to be uniform, the time taken to travel from A to B being 10 sec. What velocity is added during the time ? Find also the distance between the two posts.

20. A train is moving at the rate of 44 ft./sec. on a horizontal track when the steam is shut off. It is brought to rest by a *uniform retardation* of 4 ft./sec.^{2} Find the space passed over by the train after the steam is shut off.

21. A particle starting from rest travels with a uniform acceleration of 10 cm. sec.$^{-2}$ for 10 sec. Then with the velocity acquired moves further for next 10 sec. Finally it is again given an acceleration of 6 cm. sec.$^{-2}$ with which it moves on further. What space does the particle cover in a total period of half a minute ?

22. A body starts from rest, and moving with nniform acceleration, covers a distance of 1000 cm. in the first 10 sec. Find the distance it would travel during the 18th second of its motion.

Since $s = \frac{1}{2}\,a\,t^2$, $1000 = \frac{1}{2}\,a\,10^2 \ \therefore\ a = 20.$ cm./sec.^{2}

Again $s_{18} = \frac{1}{2} \times 20 \times 18^2$ space covered in 18 sec.

And $s_{17} = \frac{1}{2} \times 20 \times 17^2$,, in 17 sec.

$\therefore$ the space covered in the 18th second is

$s_{18} - s_{17} = \frac{1}{2} \times 20(18^2 - 17^2) = 350$ cm.

23. A particle starting from rest moves with a uniform acceleration. If it covers a distance of 9 ft. during the 5th second of its motion, find its acceleration and also the velocity at the end of 5 seconds.

24. A particle, moving with a uniform acceleration, describes 108 ft. and 140 ft. in the fifth and the seventh second of its motion respectively. Find its acceleration, and the initial velocity.

The distance travelled in

(i) 4 sec. $s_4 = v_0 \times 4 + \frac{1}{2}a \times 16$

(ii) 5 sec. $s_5 = v_0 \times 5 + \frac{1}{2}a \times 25$

Hence the distance travelled during the 5th sec.

$$s_5 - s_4 = v_0 + \tfrac{1}{2}a \times 9$$

Similarly the distance travelled during the 7th sec.

$$s_7 - s_6 = v_0 + \tfrac{1}{2}a \times 13$$

$\therefore \quad v_0 + \frac{1}{2}a \times 9 = 108$

and $\quad v_0 + \frac{1}{2}a \times 13 = 140$

Hence solving the above equations we get

$a = 16$ ft./sec.^{2} and $v_0 = 36$ ft./sec.

25. What is the value of g at a place, where a ball falls freely from rest through a vertical height of 256 ft. in 4 sec. ?

26. Find the depth of the surface of water* in a well if a stone dropped from above is found to strike the water in 3 sec. Neglect the time taken by the sound to reach the ear.

27. A juggler wants to keep 3 balls in motion by throwing each in turn to a height of 9 ft. from one hand. How many times should he move his hand up and down per minute ?

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*Take $g = 32$ ft./sec.^{2} *or* 980 cm./sec.^{2} unless stated otherwise.

Hint: The juggler moves his hand thrice during the time the ball takes to be back in the hand. The ball takes 1·5 sec.

28. Two balls which were dropped from the windows of the two different storeys of a building reached the ground at the same instant. One took one second to fall while that from the upper window took two seconds. Where was this ball when that from the lower window was let fall? What should have been the difference between the heights of the windows?

29. A photographer, wishing to time the shutter of the camera, hung a tape line from a first storey window, and had a servant drop a bullet from the zero mark on the tape. He photographed the falling bullet and the photograph showed that while the shutter was open, the bullet had moved from opposite the mark 16 ft. to the mark 16 ft. ·4 in. Find the time of exposure of the plate. ($g = 32$ ft. sec.$^{-2}$)

30. A ball is thrown vertically downwards from the top of a tower with a velocity of 5 ft./sec. Find the height of the tower, if it takes 5 sec. to reach the ground.

Since $v_0 = 5$ft./sec. and $t = 5$ sec., while $\alpha = 32$ft./sec.^{2}. we have, from $s = v_0 t + \frac{1}{2}at^2$, the height of the tower

$$s = (5 \times 5 + \tfrac{1}{2} \times 32 = 5^2 =)\ 425 \text{ ft.}$$

31. A bullet, fired vertically downwards from the top of a cliff 1064 ft. high, is found to hit a cobra near the foot of the cliff in 2 sec. Find the muzzle velocity of the bullet.

32. A balloon is rising with a velocity of 32 ft./sec. A pice is dropped from it when at a height of 768 ft. How long does it take to reach the ground after leaving the balloon?

The pice has upward velocity of 32 ft./sec. of the balloon and rises up till $v = 0$. This vertical height s is 16 ft. (from $0 = 32^2 - 2gs$) and the time in this act is 1 sec. as given by $16 = 32 \times t - \frac{1}{2} g \times t^2$.

Now the pice falls from rest through a total vertical distance of (768+16=) 784 ft. in time t. Hence, we get, from $\frac{1}{2} \times 32 \times t^2 = 784$, $t = 7$ sec. So, the total time taken by the pice to reach the ground after it is let fall is (7+1 =) 8 sec. *Or it may be solved as under.*

The upward velocity of the pice is 32 ft./sec. while its downward acceleration is g. Taking upward direction positive in the relation : $s = v_0 t + \frac{1}{2} at^2$, we get, $-768 = 32t - \frac{1}{2} \times 32 t^2$, $\therefore$ $t = 8$ sec., discarding the negative value of t. (Here velocity due to g is negative.)

In such problems if the distances above the point of projection are taken as positive, those below that point should be considered negative. Some authors consider g to be positive and hence the downward direction positivet Directions measured upwards, then, are negative.

33. A pencil, slipped off from the hand of a mechanic in a balloon, reached the ground in 10 sec. How high was the balloon, if ascending with a velocity of 16 ft./sec. when the pencil left it ?

34. A ball is let fall on the ground from an ascending balloon, when it is at the height of 4896 ft. from the ground. If the ball reaches the ground in 18 sec., determine the velocity with which the balloon is ascending.

35. A bullet was fired horizontally from the top of a tower 256 ft. high with a velocity of 2000 ft,/sec. At the same time the released cartridge-case fell to the foot of the tower. Which reached the ground first ? How far

apart were the bullet and the case when they hit the ground ? [B. A. 1923]

The bullet has two velocities. Its vertical downward velocity due to g is the same as that of the case ; hence both reach the ground at the same instant. Time taken to fall through 256 ft. by each is given by $\frac{1}{2}gt^2 = 256$. Hence $t = 4$ sec. and the horizontal distance travelled by the bullet in this time is ($2000 \times 4 =$) 8000 ft. The case falls vertically below and the bullet 8000 ft. away from it on the ground.

36. An aeroplane travelling at 90 mi./hr. has to drop mails at a certain place over which it passes. When and where must the bags to released, if the aeroplane is (1) 256 ft., and (2) 576 ft. from the ground ?

Hint : The bag retains the horizontal velocity of the aeroplane and acquires the downward vertical velocity due to g.

37. A bombarding air-plane is flying horizontally at 45 mi. an hour in still air at a height of 1296 ft. At what distance in advance of the target must the bomb be let go in order to score a bull's eye ?

38. A penny is dropped from a balloon 400 ft. high above the earth's surface. The balloon is totally immersed in, and is carried along with a horizontal current of air moving at the rate of 54·54 mi./hr. Sketch the path of the penny and find when and at what distance from the starting point it will reach the earth. Determine the direction in which it appears to move to an observer in the balloon. [I.Sc. 1928]

*39. A tennis ball is served from a height of 7 ft. and just clears the net 3 ft. high. The horizontal distance

of the server from the net is 39 ft. Find the initial velocity of the ball, and the distance of the point where it strikes the ground.

The ball travels 39 ft. in the time it falls through 4 ft. under the effect of g.

$\therefore\ 4 = \frac{1}{2} \times 32 \times t^2 \quad \therefore\ t = \frac{1}{2}$ sec. Hence the horizontal velocity imparted to the ball is ($39 \times 2 =$) 78 ft./sec. Again total time to reach the ground is given by $7 = \frac{1}{2} \times 32 \times t^2 \quad \therefore\ t^2 = \frac{7}{16}$ or $t = 0{\cdot}6615$ sec., therefore, the required distance is ($78 \times 0{\cdot}6615 =$) 51·6 ft.

***40.** An arrow shot horizontally at the bull's eye of a target 100 yds. away strikes 4·5 ft. below the bull's eye. Assuming the frictional forces of air to impart a retardation of 7 ft. sec^{-2}, calculate the initial speed of the arrow.

c. PROJECTILES **

When a body is projected with a velocity V making an angle θ with the horizontal, then

(i) the time of flight $= \dfrac{2V \sin \theta}{g}$,

(ii) the horizontal range $= \dfrac{V^2 \sin 2\theta}{g}$,

(iii) the maximum height $= \dfrac{V^2 \sin 2\theta}{2g}$,

41. A ball was thrown vertically upwards and was caught again after 10 sec. What was its original velocity? How high did it go? Take $g = 32$ ft./sec.^{2}

Hint: Total time of flight is given. Find the initial velocity and hence the maximum height. Here the angle of projection is 90°.

** Some sections are marked with double asterisks. They may be omitted by an average student.

42. A bullet is projected vertically upwards with a velocity of 192 ft./sec. Determine its velocity at the end of the 4th sec. When will the bullet come to stop ? Find also the maximum height attained by it.

The moment, the bullet leaves the gun, it gets subjected to the acceleration due to gravity in the opposite direction, which reduces its initial upward velocity finally to zero.

Hence its resultant upward velocity of the end of the 4th sec. $v_t = (192 - 32 \times 4 =)\ 64$ ft./sec.

The bullet will have zero velocity when $v_0 = gt$. *i e.* when $t = (\frac{192}{32} =)\ 6$ sec.

The bullet will come to a stop, 6 sec. after the start. Hence the maximum height is given by $h = v_0 \times t - \frac{1}{2} g \times t^2$

$\therefore \quad h = (192 \times 6 - \frac{1}{2} \times 32 \times 36 =)\ 576$ ft.

[*Or* use the relations (i) and (ii) shown above.]

43. A lump of lead is thrown horizontally from the top of a tower 80 ft. high with a velocity of 50 ft./sec. At what distance from the tower will it strike the level ground and how long will it take ? If it were thrown up at an angle of 30°, how long would it take to strike the ground ? [F. E. 1920]

Hint : As the angle of projection is 30°, if its initial velocity is V the vertical velocity is $V \sin 30° = 25$ ft./sec. Find t to fall through 80 ft. as in example 32.

44. A body is projected at an angle of 30° to the horizontal. If it passes horizontally over a wall in 4 sec., find its initial velocity.

Hint : The vertical velocity at the end of the 4th sec. is zero, therefore, $V \sin 30° = 4g$. Find V.

45. A particle is projected at an elevation of 30° with a velocity of 128 ft./sec. What will be the direction of its

motion after 2 sec. and it's maximum horizontal range ?

46. A shot is fired from a gun with an initial velocity of 800 ft./sec. at an elevation of 45°. Find the range of the shot on the horizontal plane through the point of projection.

Hint ; Find the time when the vertical velocity v_t of the particle is zero and hence the range. *Or* use the formula (ii).

47. A man runs on a straight path at 7·5 m./hr. He throws a ball vertically upwards so as to catch it again while still running, after going over 88 ft. along the path. Find the magnitude and the direction of the resultant velocity of the ball at the time of throw.

Hint : The ball acquires the vertical velocity with which it is thrown and retains the horizontal velocity of the man at the time of throw. First find the time of flight from the given horizontal velocity and the distance, hence find the vertical velocity. The resultant of these two velocities gives the required answer. Refer to formula (i) above.

2. Laws of Motion

(*a*) The change of momentum of a body is proportional to the impulse of the force applied to it, and this change takes place in the direction in which the force* acts. *i.e.* $mv_2 - mv_1 \propto ft$ with the usual notation *or*

$$\frac{mv_2 - mv_1}{t} = f, \text{ i.e. } m\alpha = f \text{ or } \alpha = \frac{f}{m} \qquad \text{(2nd Law)}$$

* The poundal and the dyne are called absolute units of force as their values are not variable ; while the lb. wt. and the gm. wt. are called gravitational units as their values at different places depend upon the acceleration due to gravity at those places. Refer to 'units' page 2.

Note :— Impulse of a force is the product of the magnitude of the force applied and the duration for which the force acts.

(*b*) The mutual actions of any two bodies are always equal and opposite. Hence whenever a body acquires a momentum, some other body acquires an equal and opposite momentum.

(*c*) Atwood's Machine :— If T be the tension in the string and a the acceleration with which the masses M_1 and M_2 (where $M_1 > M_2$) move, then the downward force minus the tension in the string gives the value of the force causing the downward motion of the heavier mass. (The effect of the mass of the string, the mass and the friction of the pulley is neglected.)

i.e. $M_1g - T = M_1a$ for the mass moving downwards (1)
and $T - M_2g = M_2a$,, ,, upwards (2)

$$\therefore \quad a = \frac{M_1 - M_2}{M_1 + M_2} g \quad \dots\dots\dots \quad (3)$$

and

$$T = \frac{2\,M_1 M_2}{M_1 + M_2} g \quad \dots\dots\dots \quad (4)$$

48. What is the momentum of a mass of 1 lb. moving with a velocity of 20 ft. per second ?

49. What velocity will be generated in a body, weighing 100 gm., when acted upon by a force of 200 dynes for 4 sec. ?

Since

the change in momentum = the impressed force × time.

$100 \times v = 200 \times 4$. Hence $v = 8$ cm./sec.

50. If a force of impulse 216 dyne-sec. acts on a mass of 6 gm., find the momentum gained by the mass.

51. A cricket ball of mass $\frac{3}{4}$ of a lb. and moving with a speed of 30 ft./sec. is hit in such a way, that after the blow, it moves with a speed of 45 ft./sec. and in a direction at right angles to its original direction. Find the change

of velocity produced and assuming the blow to last $\frac{1}{50}$ of a second, calculate the average force with which the blow is struck.

Hint: Using the triangle of velocities, find the change in velocity v'. Hence find the required force from $mv = ft$. Refer to the example 9.

52. A railway truck of mass 10 tons, while moving with a velocity of 4 ft./sec., comes into collision with fixed buffers and is stopped in 0·5 sec. What is the average force of the blow in ton-wt.?

53. A man and his bicycle weigh 172·8 lb. While going at a speed of 15 mi./hr. on a level road, he suddenly ceases to pedal and in 22 sec. finds the speed reduced by 7·5 mi./hr. Find the force of retardation in gravitational units.

Hint: First find the quantity of motion destroyed. This equals the impulse of the force acting. The period during which this force acts is known. Hence determine the force required.

54. A shell weighing 100 lb. was fired from a gun with a muzzle velocity of 2000 ft./sec. Find in poundals (*i.e.* in absolute units) the average retarding force due to the resistance of the air which reduces its velocity to 1900 ft./sec, in 4 sec.

55. A circket ball, of mass 180 gm. and moving with a speed of 100 cm./sec, is hit in such a way that it returns with the same speed in the direction just opposite to the original one. Find the change of velocity produced and assuming the blow to last 0·02 sec. calculate the average force with which the blow is struck.

Hint: To find the change in velocity consider the proper algebraic signs from the directions of velocities.

56. In what time will a velocity of 45 mi./hr. be generated in a train of 192 tons, if it starts, from rest and the pull exerted by the locomotive is 4 tons-wt.?

57. A force of 10 gm. wt. required 0·5 sec. to stop a body which was moving with a uniform velocity of 140 cm./sec. Find the weight of the body.

58. A train starts from rest and is found to gain a velocity of 30 mi./hr. in 88 sec. Find the acceleration generated and the pull exerted by the locomotive, if the mass of the train is 192 tons.

Since the velocity is 30 mi./hr., it is 44 ft./sec. But this velocity is gained in 88 sec. Hence the acceleration $a = (44/88 =)$ 0·5 ft./sec.^{2} Again the mass of the train is $(192 \times 2240 =)$ 430080 lbs., therefore the pull exerted, $ma = (430080 \times 0{\cdot}5 =)$ 215040 **poundals.**

59. The last waggon in a train weighs with its load 10 tons and the train is moving with an acceleration of 2 ft./sec.^{2}. Find the force in the couplings in tons-wt. Neglect friction.

60. A freight car, weighing 10 tons, is drawn by a steady pull of 224 lb. wt. acting for a minute. What acceleration is imparted to the car? Find the velocity also at the end.

61. A small trolly on a horizontal track weighing 450 kg. is moving uniformly; when the driving power is cut off It comes to rest after moving over 100 m. without the use of any brakes in 40 sec. Find the value of the frictional force in dynes which it has to overcome, assuming it to be constant throughout. [I. Sc. 1926]

Hint: The retarding force produces negative acceleration. The acceleration is given by the ratio $2s/t^2$.

62. Two men, each exerting a constant force of 60 lb. wt., set a waggon weighing 6 tons in motion. The frictional resistance amounts to 10 lb. wt. per ton. Calculate the distance through which the waggon is moved in a minute.

Hint: The moving force in this case is equal to the force applied minus the total force of friction. Calculate the acceleration and hence the distance.

63. Find the average retarding force of the brakes when a train moving with a velocity of 45 mi./hr. has its speed reduced to 30 mi./hr. in 1210 ft. The mass of the train is 176 tons.

64. A force equal to the weight of 1 kg. acts on a body (at rest) for 10 sec. and causes it to describe 10 m. in that time. Find the mass of the body.

Hint: Find the acceleration from the space travelled and the time. The moving force is given, determine the mass.

65. A constant force of 50 lb. wt. imparts a uniform acceleration of 2 ft./sec.^{2} to a body of mass 400 lb. Find the frictional resistance acting on the body.

The moving force of $50\,g = f$ poundals where f is the force of friction. While the mass moved is 400 lb. Hence the acceleration $2 = (50 \times 32 - f)/400 \quad \therefore \quad f = 800$ poundals.

66. What time will a paper pellet, weighing 0·5 gm., take to fall through a vetrical height of 16 m., if the average resistance acting on it due to air is 90 dynes ?

67. A weight of 50 lb. is moving at a speed of 15 ft./sec. and is acted upon for 20 sec. by a force of 20 lb. wt. in the direction of its motion. What is the distance moved through during the time. [F.E. 1916]

Hint : The body moves further with an acceleration in addition to its initial velocity. The acceleration is given by the moving force divided by the mass moved,($F=M\alpha$), Hence determine the space moved over in 20 sec.

68. A mass of 120 lb. is pushed up a smooth incline rising 11 in 120 by a force of 26 lb. wt. Find the acceleration generated and the space described in 4sec.

Hint : Moving force along the inclined plane is the force applied F minus the retarding force $W \sin \theta$ on the plane due to the weight of the body. Mass moved is 120lb.

69. A body slides, from rest, down a frictionless inclined plane rising 5 in 16. Find the distance travelled by it in 10 sec. and its velocity in the end.

Hint: As the direction of g is vertically downwards, the acceleration with which the body moves along the plane is given by the component of g parallel to the plane.

70. A body is projected up a smooth inclined plane with a velocity of 40 ft./sec. Find the time when it will cease to rise up. The angle of inclination of the plane is 30°.

Hint: Here the motion is opposed by the component of g acting along the plane. Hence find the time during which the initial velocity gets reduced to zero.

71. A mass of 16 lb., lying on a smooth horizontal table 25 ft. from its edge, is drawn along the surface by a mass of 4lb. hanging freely. How long does it take to reach the edge ?

> Here the moving force is ($4\times32=$) 128 poundals while the mass moved is ($16+4=$) 20 lb., therefore the acceleration is 6·4 ft./sec.^{2} from $F=m\alpha$; and the time taken by the mass to move through a distance of 25. ft. is 2·8 sec. from $s=\frac{1}{2}\chi t^2$.

72. A 6 lb. mass is dragged along on a horizontal

surface of a smooth table by a mass of 4 lb. hanging freely at its edge. Find the tension in the connecting string and the distance moved in 2 sec.

Hint; Find the acceleration. The moving force is $(4 \times 32 -)$ 128 poundals and the mass moved $(6+4+)$ 10 lbs. Consider the forces acting on the mass of 4 lb. The force with which the mass moves down is the difference between its weight and the tension in the string. Hence find the tension.

73. A mass of 10 lb. hanging freely draws an equal mass up a smooth inclined plane through a distance of 12 ft. in 2 sec. Find the inclination of the plane to the horizontal.

From $s = \frac{1}{2}at^2$, we get $12 = \frac{1}{2}a \times 2 \times 2$, $\therefore$ $a = 6$ ft./sec.^{2}
Now the two forces acting on the system are the weight of 10 lb. and the component along the plane of the other weight of 10 lb. They oppose each other.
$\therefore$ the moving force $= 10 \times 32 - 10 \times 32 \sin\theta$, but

$$\text{acceleration} = \frac{\text{moving force}}{\text{mass moved}} \therefore 6 = \frac{10 \times 32 - 10 \times 32 \times \sin\theta}{10+10},$$

Hence $\sin\theta = \frac{5}{8}$ and $\theta = 38{\cdot}7^\circ$

***74.** A certian mass hanging freely draws another mass of 20 lb. up a smooth inclined plane of the angle of inclination 30°. Find the value of the mass, when the acceleration is (1) 0, (2) 2 ft./sec^2. Also determine the tension in the string in each case.

Hint: Retarding force on the inclined plane due to the weight of the body plus the force to move the body up the plane equals the tension in the string and is given by $T = M_2 g \sin 30^\circ + M_2 a$ *or* $T = M_1 g - M_1 a$, where $M_2 = 20$.

75. Two masses of 400 gm. each are suspended from the two ends of a string passing over a light and smooth

pulley. The rider on one of the masses is 10 gm. Find the tension on the string and the distance through which the mass with the rider discends in 2 sec. from its start. ($g=981$ cm./sec.^{2}) [B.A.1928]

$M_1=400+10$ gm. $M_2=400$ gm. Substituting the known values in $\alpha=\frac{M_1-M_2}{M_1+M_2}g$, we get $a=\frac{10\times 981}{810}=\frac{981}{81}$ cm./sec.^{2}, hence the distance moved in 2 sec. is $\frac{1}{2}\times\frac{981}{81}\times 4=24\frac{2}{9}$ cm. Again the tension T is given by the relation $M_1g-T=M_1\alpha$ or $T-M_2g=M_2a$. $\therefore$ $T=404\cdot 9$ gm. wt.

76. Two masses connected by a string hanging over a smooth pulley move over 16 ft. from in 2 sec. If the heavier weight be a mass 5oz. find the other.

77. A thread, passing over a grooved pulley which is free to rotate, has equal weights attached to its ends. An over-weight of 1 gm. is added on to one of them. It is found that in 5 sec. from the start the whole system has described 50 cm. Find the value of the equal weights and the tension in the string.

***78.** A rope hangs down over a smooth pulley and a man weighing 60 kg. slides down the portion of the rope on one side of the pulley with a unifrom acceleration of 140 cm./sec.^{2} Determine the weight of a man, who while sliding down the other portion of the rope with an acceleration of 168 cm./sec.^{2} is able to keep the rope at rest. What will happen, if their weights are equal?

Hint: Equte the resultant vertical downward force on both the sides *i.e.*, m $(980-168)=60$ $(980-140)$. If the weights are equal, rope will also move with an acceleration.

79. Two equal masses are attached to the ends of a string passing over a light frictionless pulley. One of the

weights is supported on a small table while the other is raised through a vertical distance of 10 cm. and allowed to fall freely through the same distance. Find the velocity of the two weights when the string becomes taut.

Hint: The momentum acquired by the falling body during the descent is shared by both the bodies. Hence find the velocity of each.

*80. Two buckets of 7·5 lb. each are supported by a rope passing over a smooth pulley and are at rest. A mass of one pound is dropped from a height of 4 ft. into one of the buckets. Find the time taken by the buckets in moving through a vertical distance of 12 ft. after the motion is imparted.

Hint: Find the initial velocity gained by the buckets (including the one pound body) from the loss of momentum of the falling mass which it had gained during its fall through 4 ft. The system then begins to move with this velocity in addition to the acceleration imparted to it by this excess of 1-lb. body on one side. Momentum of the 1-lb. body is shared by the buckets and the body itself.

81. A rifle, weighing 20 lb. discharges 1-oz. bullet with a velocity of 400 ft./sec. What will be the velocity of the rifle in the opposite direction ?

If V be the velocity of the rifle, its momentum $=20\times V$ and the momentum of the bullet $=(\frac{1}{16}\times 400=)$ 25 lb. ft./sec. Since action is equal to the reaction

$$20V=25$$

$$\therefore \quad V=1{\cdot}25 \text{ ft./sec.}$$

82. A bullet weighing 3 gm., is fired from a gun weighing 10 kg. What is the muzzle velocity of the bullet, if the velocity of recoil of the gun is 15 cm./sec. ?

83. A man weighing 120 lb., gives an impulse to a

100–lb. canoe as he steps out of it. If the canoe is imparted a velocity of 2 ft./sec. calculate the velocity the man aquires.

84. A lion, weighing 500 lb. moves up and down a cage mounted on frictionless ball bearings. If the weight of the cage be 1000 lb. and the speed with which the lion moves be 6 ft./sec., find the speed of the cage in the opposite direction. What is the speed of the lion relatively to the ground ?

Hint: Momenta are equal. Find the speed of the cage in the opposite direction, hence the speed of the lion relatively to the ground. Note that the cage moves in the opposite direction taking the lion with it. Hence

$$(m+m')\ v'=mv$$

85. A cage of mass one ton is lowered down a pit with a nniform acceleration of 8 ft. sec.$^{-2}$ What is the stretching force in the rope ? With what force will a mass of 100 lb. lying on the bottom of the cage press on it ? ($g=32$ ft. sec.$^{-2}$) [I.Sc. 1934]

Hint: Downward force due to the weight of the cage minus the tension in the rope gives the moving force. Find this tension. Similarly the weight of mass minus the reaction of the bottom on it equals the moving force on the mass. Pressure equals this reaction.

86. A man weighing 120 lb. is standing in a lift. When the lift begins to go up with an acceleration of 8 ft /sec.^{2} find the pressure on the floor of the lift and the apparent weight of the man. [B.A. 1925]

Hint: Here the lift rises up. Hence the tension is greater than the downward force; the resultant being the moving force. The apparent weight of the man equals the pressure due to his mass on the platform.

87. In the example 86, find the pressure on the floor if the lift descends with an acceleration of 8 ft./sec^2.

88. A person in a balloon finds that his spring balance shows a weight of 12·5 lb. when a mass of 10 lb. is suspended from it. What is his motion? Find the height he will rise in 15 sec. from the start. [I.Sc. 1927]

Hint: The balance not only supports the body but also gives it an acceleration. This acceleration is given by the extra force of 2·5 lb wt. indicated by the balance. Mass moved is 10 lb., find the acceleration and hence the height.

89. An elevator is ascending with a uniform acceleration of 8 ft. sec.$^{-2}$. A 100-lb. body is suspended from a spring balance in it. What will be the reading on the balance?

90. A person who is just strong enough to lift 140 lb. can lift 160 lb. from the floor of a descending elevator. What is the acceleration of the elevator? If it rises with (1) the same acceleration as above, (2) a uniform velocity, find the amount of mass he will be able to lift in each case.

When a lift is descending with an acceleration, the weight of a body is decreased. Here the weight of the body raised by the man should not exceed $(140 \times 32 =)$ 4480 poundals. If a be the effective acceleration of gravity on the mass in the balloon then $160a = 140 \times 32$ $\therefore$ $a = 28$ ft./sec^2. $\therefore$ the balloon must be going down with an acceleration of $(32 - 28 =)$ 4 ft./sec^2. or is moving with an acceleration of $(28 - 32 =) -4$ ft./sec.^{2} (1) The effective acceleration on the mass will be $(32 + 4 =)$ 36 ft./sec^2. If M be the mass that can be lifted, then $36\,M = 140 \times 32$

$\therefore \quad M = 124\frac{4}{9}$ lb. (2) There is no acceleration, hence the man can lift a mass of 140 lb.

*91. A seconds-pendulum carried in an elevator makes 45 complete oscillations in 100 sec. What is the acceleration of the elevator ? State whether it is ascending or descending.

Hint : Seconds-pendulum has a period of 2 sec. The acceleration responsible for the oscillation is equal to g minus the acceleration with which the point of suspension moves towards the earth.

3. Composition and resolution of forces.

(i) The resultant of two or more parallel forces acting in the same direction is equal to the sum of these forces.

(ii) The resultant of two oppositely directed forces applied at the same point is equal to the difference between them, and the direction of it is that of the greater forces

(iii) If two forces P and Q act at a point and subtend an angle θ between them, the resultant R is given by $R^2 = P^2 + Q^2 + 2PQ \cos \theta$.

(iv) If three forces acting on a particle keep it in equilibrium, each is proportional to the sine of the angle between the other two *or* as these forces form a close triangle ABC, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \text{Lami's theorem.}$$

92. Two pulls of 6 lb. wt. and 12 lb. wt. respectively are acting at a point in the same direction. Find their resultant.

93. Two forces are acting at a point in opposite directions. If their magnitudes are 10 and 12 lb.wt.respectively, determine the magnitude and the direction of the resultant,

The resultant $R = P+Q$. Taking one direction as positive while the other as negative we get

$$R = (12-10=)+2 \text{ lb. wt.}$$

The $+ve$ sign shows that the direction of the resultant is the same as that of the greater force *viz.* 12 lb. wt.

94. A picture, weighing 20 lb. is hung by a cord passing through two rings screwed at the upper corners of the frame. The cord is 4 ft. long and the rings 3 ft. apart. Find the tension in the cord, if the picture is suspended from the two pegs in a horizontal line.

Hint: The force of 20 lb.wt. is equally shared by the pegs. Hence find the tension.

95. A body is acted upon simultaneously by two forces, one of 60 lb.wt. directed Northwards, while the other of 80 lb.wt. directed Westwards. Find the magnitude and the direction of the resultant force.

Draw OA and OB to scale to represent the given forces of 60 lb, wt. and 80 lb. wt, respectively. Complete the parallelogram $OACB$ on OB. Join the points O and C. The diagonal OC gives the magnitude and direction of the resultant to scale. *Or by calculation,*

since the $\angle AOB = 90°$, the *resultant* OC is given by $OC^2 = OB^2+BC^2$, using the relation $R^2 = P^2+Q^2+2PQ\cos\theta$.

$$\therefore \quad OC = (\sqrt{60^2+80^2} =)\ 100 \text{ lb. wt.}$$

and its direction is given by the relation $\tan\phi = \frac{80}{60}$

$$\therefore \quad \phi = 53{\cdot}1° \text{ to the west of North.}$$

96. A horse exerts a pull of 250 lb. wt. on a tram car ht 30° to the direction of the rails. Find the force urging the tram forward and that tending to lift it off the rails.

Hint: Here the vertical component of the applied force acts upwards trying to lift the car up while the horizontal one is dragging the car forwards.

97. A man pulls a nail by means of a string inclined at 30° to the wall. If he exerts a pull of 30 lb.wt. find the force tending to draw the nail out.

98. A mass of 500 gm. is suspended by a string from a nail. What horizontal force is required to displace it until the string makes an angle of 30° with the vertical? Calculate the force on the peg.

Three forces are acting at the point where the horizontal force is applied. The vectors of these forces should form a closed triangle. If P be the horizontal force and as Q the vertical force $= 500$ gm. wt., $P = Q \tan 30° = 288{\cdot}7$ gm.wt. The force on the peg equals the tension in the string and is equal to the resultant R of P and Q

$$\therefore \quad R = \frac{500}{\cos 30°} = 577{\cdot}4 \text{ gm. wt.}$$

99. Find the force required to keep a slab of one ton on a smooth inclined plane, if the slope† is 75% and the force is applied parallel to the plane.

100. An inclined plane rises 1 in 10. What force acting up the plane and parallel to it will just sustain a mass of 200 lb. on the incline? Neglect the friction between the surfaces.

101. A carriage, mounted on frictionless wheels, is resting on an inclined plane of grade† 50%. If the weight of the carriage is 20 lb., find the force required to keep it in equilibrium (i) when it acts parallel to the plane, (ii) when it acts horizontally.

† The *grade* of an incline means the ratio of the height to the length of the incline and is commonly expressed in the percentage form. The slope means the ratio of the height to the base.

102. A body of weight 30 lb. is in equilibrium on a smooth plane inclined at an angle of 45° to the horizontal, being supported by a force P acting at an angle θ with the plane. Find P and show that $R/P = \cos\theta - \sin\theta$, where R is the reaction of the plane. [I.Sc. 1936 Math.]

103. Three cords are tied together at a point, one of these is pulled due North with a force of 6 lb. wt., and the other due East with a force of 8 lb.wt. With what force must the third be pulled in order to keep the whole system in equilibrium ?

Hint : First find the resultant of the two given forces. The force to be applied should equal this resultant in magnitude. Hence the magnitude of the resultant with the direction reversed gives the required force (*i.e.* the equilibrant).

104. Two men standing on opposite sides of a pit are drawing up a bucket of earth by means of two ropes. When the bucket is in equilibrium, one man is exerting a force of 70 lb.wt. on a rope inclined at 25° to the vertical, while the other man is exerting a force of 50 lb.wt. on a rope inclined at 55° to the horizontal. Find the weight of the bucket.

105. A child weighing 42 lb. is seated in a swing suspended by two ropes, each inclined at 30° to the vertical. Find the tension in each of the ropes. The swing weighs 10 lb.

Hint : Resolve the total downward force into two components along the two ropes.

106. A balloon is held by a rope making 30° with the horizontal. Find the tension of the rope and the horizontal pressure of the wind on the balloon, if it can support a weight of 400 kg.

Hint : Three forces act on the balloon : tension along the string, wind pressure, and the upthrust of 400 kg. wt.

vertically upwards. Draw the triangle of forces. Use Lami's theorem.

***107** An oil-painting is hung on a wall from an iron peg by means of a cord passing over it, the two ends being tied to the two rings 25 inches apart in the top of the frame which is horizontal. The peg is 20 inches above the top. If the frame weighs 50 lb., find the tension in the cord.

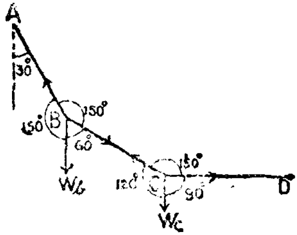
Hint: The angle between the vertical and one of the cords can be obtained from the relation $\tan\theta = \frac{25}{2\times20}$. Hence component of T (the tension in one of the cords) along the vertical is $T\cos\theta$. But the total force of 50 lb. wt. is balanced by a total of $2T\cos\theta$. Hence determine T.

***108**. A picture weighing 2 lb. is hung by a cord fastened to the two upper corners of its frame and passing over a peg. The parts of the cord at the two sides of the peg are inclined to one another at an angle of 60° keeping the upper and lower edges of the frame horizontal. Find the tension in each part of the cord.

Hint: Three forces are acting at the peg. Tension in the side strings and the reaction in the peg. Refer to the Ex. 106.

***109**. $ABCD$ is a light string tied to two fixed points A and D and is carrying weights at B and C. Determine, the ratio of these weights, if AB is inclined at 30° to the vertical, BC at 30° to the horizontal and CD is horizontal.

[F. E. 1920]



Hint: Consider the equilibrium of B by the triangle of forces. The weight B is equal to the tension along BC. Similarly for the equilibrium of C, the tension along BC is double the weight of C Use the relation iv.

4. Moments of Forces

(*a*) The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

(*b*) The moment of a couple is equal to the product of one of the forces and the perpendicular distance between their lines of action.

110. Find the magnitude and the position of the resultant of two forces 10 and 15 lb. wt. respectively, acting at the ends of a light pole 20 ft. long.

Since the forces are acting in the same direction the resultant = ($10+15=$) 25 lb. wt.

Again of the distance of the point of application of the resultant be a from the force of 10 lb. wt. and b from the force of 15 lb.wt., taking moments about this point we have, $10\times a-15\times b=0$ and $a+b=20$

$$\therefore \quad a=12 \text{ ft. and } b=8 \text{ ft.}$$

The resultant acts at a point 12 ft. from the 10 lb.wt.

111. A pole, 20 ft. long, is carried by two men, each end resting on the shoulder of a man. A mass of 140 lb. is suspended from the middle. Find the weight carried by each, neglecting the weight of the pole.

112. A uniform plank AB, 40 ft. long and weighing 100 lb. rests on supports at its ends. A weight of 500 lb. is suspended from a point on it at a distance of 8 ft. from A. What is the pressure on each of the supports ?

Hint : The total downward force pressing the supports is ($100+500=$) 600 lb. wt. Take the moments of the forces about one end of the plank, and find the magnitude of the pressure at the other end.

113. The horizontal roadway of a bridge is 36 ft. long and weighs 6 tons. The bridge rests on similar supports

at the ends. Determine the pressure on each support when a carriage weighing two tons is (1) halfway, (2) one-forth of the way across.

114. *AB* is a uniform bar pivoted at its mid-point *C*. A weight of 5 lb. is suspened from one of the arms from a point, distant 15 inches from *C*. Where should a 3 lb.wt. be applied to the bar to restore equilibrium ?

115. A 154-lb. man sitting on one end of a see-saw is balanced by a 56-lb. child sitting on the other end. The length of the see-saw is 12 ft. and its mass 250 lb. What is the position of the point of support ? Assume the beam to have a uniform linear density.

116. A uniform beam 24 ft. long and weighing 240 lb. is resting upon two supports one 6 ft. and the other at a distance of 16 ft. from the same end. What is the pressure on each prop when a man weighing 200 lb. stands as near the other end as he can without upsetting the beam ? Find his distance from the second prop.

Hint : The beam will just tilt about the prop nearest to the man, and hence the pressure on the other will be zero.

117. A pulley, 2 ft. in diameter, is acted upon by a turning moment of 240 lb. in. Find the force of friction which while acting round the rim would produce balance.

118. Two boys, each exerting a force of 10 lb. wt., rotate a horizontal beam, 6 ft. long, pivoted at the mid-point. Find the moment of the couple in gravitational and absolute units. The forces act at the ends and at right angles to the length of the bar.

119. What will be the couple in example 118, if the forces are parallel and inclined to the length of the bar at 30° ?

120. A water-pot weighing 40 lb. is to be raised from a well by a wheel and axle. The radii of which are 20 in. and 4 in. respectively. Find the force that must be applied to the wheel arm and the pressure on the supports.

Hint: The moments of the two forces acting along the two circumferences are equal.

121. A capstan has five spokes projecting 8 ft. from the axis. The cylinder round which the rope is wound has a diameter of 7 in. If five men exert a pull of 70 lb. each at the end of the spokes, find the tension in the rope.

Hint: The moments about the *axis* are equal. A spoke is a force-arm. The length of the arm is 8 ft.

122. The weight of an anchor is 3·6 tons. If the radius of the axle of a capstan round which a rope is wound be 4 in., and to raise the anchor four sailors are required to exert a force of 112 lb. each at the end of the spokes, find the length of each spoke.

‡**123.** A cylinder 6 cm. in height and having a base radius 2·25 cm. is placed on an inclined plane in a vertical position. Find the inclination of the plane when the cylinder just topples over.

Hint: The vertical line passing through the centre of gravity of the cylinder must also pass through a point just a little beyond the circumference of the base in the inclined position. Tan $\theta = 2{\cdot}25/3{\cdot}00$.

124. An iron rod 60 cm. in length and weighing 2·5 kilos balances on a knife edge put under it at a distance of 25 cm. from the end *A*, when a weight of one kilogram is suspended from the same end. Find the position of the centre of gravity of the rod.

‡Though the theory on centre of gravity is not in the syllabus, the examples of fundamental importance which could be worked out with the help of the portion studied in the High Schools are given here.

Let x be the distance of *C.G.* from the knife-edge, then taking the moments about the point of support, we get

$$25\times 1 = x\times 2{\cdot}5 \quad \therefore\ x = 10 \text{ cm. } or \text{ } 35 \text{ cm. from } A.$$

125. A wooden pole AB, 20 ft. long, balances on a peg put under it at a distance of 12 ft. from the end A. It also balances on a peg put under its centre, when a weight of 10 lb. is placed at the same end A. Find the weight of the pole.

Hint : In the first case the position of the centre of gravity is ascertained. It is 2 ft. from the centre on the side of B. The weight of the pole acts through the centre of gravity and this is balanced by the force of 10 lb. wt. acting at A.

126. A uniform metre-rod AB is freely suspended at a point 10 cm. from the end A. A load of 80 gm. attached at a distance of 2 cm. from the same end is just able to keep the rod horizontal. Determine the weight of the rod.

127. A uniform rod AB 3 ft. in length and weighing 10 lb. is placed horizontally on two supports at a distance of 10 in. Find the position of the support from the end A, if the pressure difference on the supports is 4 lb.wt.

Let the supports be P and Q and the pressure on them p and q respectively. Let the distance of P from the end A be x in., then that of $Q = x+10$ in. By data, $p-q = 4$ lb.wt. but $p+q = 10$ lb.wt. $\therefore$ $p = 7$ lb.wt. and $q = 3$ lb.wt.

Again the weight of the rod acts vertically downwards through its *C.G.*, hence taking the moments of the forces about the point A we get $7x+3(x+10) = 18\times 10$

$$\therefore \quad x = 15 \text{ in. and } x+10 = 25 \text{ in.}$$

5. Friction

The coefficient of friction on a plane surface

$$= \frac{\text{force of friction between the surfaces}}{\text{normal pressure acting on the surface}}$$

$\mu = \frac{f_\circ}{W}$ for a horizontal surface

But $\mu = \frac{f^\circ}{W\cos\theta}$ for a surface making an angle θ with the horizontal.

Also $\mu = \tan\theta$, where θ is the limiting angle *or* the angle of repose as the case may be.

128. The coefficient of friction between the two sur faces is 0·3. Find the force of friction, if the normal pressure is 100 lb.wt.

Since force of friction $= \mu \times$ normal pressure

$$f = 0{\cdot}3 \times 100$$

$$= 30 \text{ lb., wt. } or \text{ } 960 \text{ poundals.}$$

129. A force of 30 lb. wt. is required just to push a trunk weighing 200 lb., along the floor of a ware-house. Find the coefficient of friction between the surface of the trunk and the floor.

Hint: The force applied just overcomes the force of friction, hence it is equal in magnitude to the friction.

130. What force does a horse exert on a level ground in pulling a cart weighing one ton? The cart possesses no wheels and the coefficient of friction between the surface of the cart and the road is 0·2.

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*Portion which is not in the syllubus is of fundamental importance to a First Year and P. U. Student, is included with an asterisk mark at the begining of the section. These problems can be solved by an average student with the help of the High School Physics.

131. A stone brick, weighing 100 lb., rests on a rough ground. Find the force applied parallel to the ground to just move it. ($\mu = 0{\cdot}2$.)

132. A railway porter uses a hand-cart for carrying some luggage weighing 144 lb. If the poles of the cart are inclined to the horizontal at an angle of 23°, find the force required to start it moving. How much force would have been saved, if he were to start the cart with horizontal force? $\mu = 0{\cdot}23$ and $\cos 23° = 0{\cdot}92$.

Hint: First determine the frictional force. The porter has to exert a horizontal force to just overcome this friction. The angle between the poles and the horizontal is given, hence determine the force along the poles.

133. A body weighing 120 lb. rests on a rough horizontal platform and is just being moved along by a force of 40 lb.wt. making an angle of 30° with the horizontal. Find the force of friction and the coefficient of friction.

Hint: The forces acting on the body are three (i) its weight, (2) the force of friction and (3) the force applied. Draw the triangle of forces. The horizontal component of the force applied is equal to the force of friction. The pressure on the surface is equal to the weight of the body minus the vertical component of the force applied.

134. A heavy truck with a plane bottom is lying on a rough horizontal plane. It is acted upon by a force inclined at 30° to the horizontal and the force is increased till the body just begins to slide. Compare the force with the load to be moved. ($\mu = 0{\cdot}3$).

6. Work and Energy.*

When a force F acts on a body and thereby its point of application moves through a distance S in the line of

action of the force, the work done is given by the product of the force and the distance, $W = F \times S$ (i)

$$\text{Power} = \text{Rate of doing work} = \frac{\text{Force} \times \text{distance}}{\text{time}}$$

One horse-power ($H.P.$) = 746 watts *or* 550 ft lb./sec.

The potential energy ($P.E.$) of a body is the energy due to its position and is given by $m \times g \times h$. (ii)

The kinetic energy ($K.E.$) of a body is the energy due to its motion and is given by $\frac{1}{2}mv^2$; where m,v,g, and h have usual meanings. (iii)

Work done by a couple C in rotating a system through an angle of θ radians $= C \times \theta$. (vi)

135. How much work is spent in moving a truck at a uniform speed of 10 ft./sec. against a resistance of 40 lb.wt. for 10 sec.? Find the rate at which the work is done.

Distance S through which the truck moves in 10 sec. against a force of 40 lb. wt. is ($10 \times 10 =$) 100 ft. Hence the work done against this resistance $= (40 \times 100)$ $= 4000$ ft. lb. Again the rate of doing work $= (4000/10 =)$ 400 ft.lb./sec.

136. A farm tractor draws a plough at a speed of 2·2 ft./sec. Find the horse-power at which the tractor is working, if the average pull exerted by it is 1000 lb. wt.

Hint : First find the work done (in foot-pounds) per sec. by the pull exerted by the tractor, hence the $H.P.$

137. A cyclist goes over a distance of 7·5 miles in one hour. If the resistance offered by the road is 7lb. wt. find the $H.P.$ at which he works.

138. A horse swims across a river 50yd. wide in 2 min. Supposing that it exerts one horse-power in swimming, find in gravitational units the resistance offered by the water.

139. An aeroplane is travelling horizontally at 37·3 m./sec. and is driven by an engine developing 50 *H.P.* Find in dynes the force of the backward thrust on the propeller.

140. A locomotive, weighing 100 tons, draws a train of carriages weighing 400 tons, at a speed of 30 mi./hr. If the friction between the rails and the wheels of the train is 10 lb per ton, find the *H.P.* developed at the driving wheels.

Hint: Total load is 500 tons. Hence the friction is 5000 lb. wt.

141. A man draws a bucket of water, weighing 75 lb., from a well 100 ft. deep. How much energy does he spend?

Here the work done by the man is used in increasing the *P.E.* of the bucket. The *P.E.* of the bucket is increased by ($75 \times 32 \times 100 =$) 240000 ft. pdls. ($m \times g \times h = P.E.$)

$\therefore$ the energy spent by the man = 240000 ft. pdls.

Alternative method :-

The downward force due to the weight of the bucket = ($75 \times 32 =$) 2400 poundals.

The bucket is raised through a height of 100 ft. against this force, hence the energy spent by the man = ($2400 \times 100 =$) 240000 ft. pdls. ($W = F \times S$)

142. A man, weighing 120 lb., carries a bicycle of mass 30 lb. upstairs, 25 ft, above the ground floor. Calculate in gravitational units the total work that he does. If he requires 10 sec. to do this work, what *H.P.* does he spend?

143. If clouds were one mile above the earth and there is a rainfall enough to cover an area of half a square mile of the surface half-an inch deep, how many foot-

pounds of work would have been spent in raising the water to the clouds ?

144. *Find the H. P.* of an engine required to pump 1000 gallons of water per sec. from a well 110 ft. deep, assuming the water level in the well to remain unchanged.

145. A cylindrical reservoir filled with water has a diameter of 42 ft. and a depth of 14 ft. Find the amount of energy the water possesses and the average *H.P.* it can develop, if the water is delivered at a depth of 110 ft. below the bottom of the reservoir and the reservoir emptied in 10 min.

Hint: The potential energy equals the work done in placing the body in the given position. The effective head of water is from the point of delivery to the mean height of the water in the reservoir.

146. A machine-gun fires 300 bullets per min. each weighing 1 oz. with a velocity of 1600 ft./sec. Find the kinetic energy of the bullets and the *H. P.* at which the gun is working.

The bullets discharged per sec. $= (300/60 =)\ 5$.

The velocity with which each

bullet leaves the gun $=$ *1600 ft./sec.*

Mass of the bullets leaving the gun/sec. $= (\frac{1}{16} \times 5 =) \frac{5}{16}$ lb.

Hence the $K.E. = (\frac{1}{2} \times \frac{5}{16} \times 1600^2 =) 4 \times 10^5$ ft. pdls.

i.e. 12500 ft. lbs. and $H.P. = (\frac{12500}{550} =) 22\frac{8}{11}$.

147. A mass of 30 kg. starts from rest and after a short time is found to be moving with a velocity of 14·4 km./hr. How much kinetic energy (in joules) does it possess ?

148. A 4-oz. bullet is shot vertically upwards. What is its kinetic energy, when it leaves the gun, if its muzzle velocity is 400 ft./sec ? How high will it rise, if all the

energy is utilised in rising up ? Neglect the effect of the earth's attraction.

Hint: The *K.E.* which the bullet possesses when it leaves the gun, utilized in raising its *P.E.*

149. A person throws half-a-pound stone vertically downwards from the edge of a cliff. It hits the ground below after 10 sec.; the initial velocity acquired by it in the act of throwing being 22 ft./sec. Find the velocity with which it strikes the ground, the height of the cliff, the momentum and the energy of the stone at the time of impact.

150. A small body of mass 150 gm. moves along a straight line and at times $t=0,1,2,3,4$ sec., it is at distances 12, 21,38,63, and 96 cm. respectively from a fixed position in that line. Find (*a*) the acceleration, (*b*) the force acting on the body, (*c*) the kinetic energies at times $t=0$, and $t=5$ sec. [I.Sc. 1924]

Using the relation $s=vt+\frac{1}{2}at^2$, we get

$21-12=v_0 \quad +\frac{1}{2}\alpha\times 1$, space covered during 1st sec.
$38-12=v_0\times 2+\frac{1}{2}a\times 4$, ,, ,, ,, 2 sec.
$\therefore \quad 17=v_0 \quad +\frac{1}{2}\alpha\times 3$, ,, ,, ,, 2nd sec. (*i*)
Again $63-12=v_0\times 3+\frac{1}{2}a\times 9$ space covered during 3 sec.
$96-12=v_0\times 4+\frac{1}{2}a\times 16$,, ,, ,, 4 ,,
$\therefore \quad 33=v_0 \quad +\frac{1}{2}\alpha\times 7$,, ,, ,, 4th sec. (*ii*)

Solving (*i*) and (*ii*) for v_0 and α we get $\alpha=8$cm./sec.^{2} and $v_0=5$cm./sec. (*b*) Hence the force on the body is $(150\times 8=)$ 1200 dynes, from $f=m\times a$,
(*c*) Velocity of the body,

(i) when $t=0$, v is 5 cm./sec. and

(ii) when $t=5$, $v_5=(5+8\times 5=)$ 45 cm./sec.

$\therefore$ *K.E.* in case (i) $=(\frac{1}{2}\times 150\times 5^2=)$ 1875 ergs.
and *K.E.* in case (ii) $=(\frac{1}{2}\times 150\times 45^2=)$ 151875 ,,

151. What force will stop, in half a mile, a train of 200 tons moving at 30 mi./hr. ? Express the result in gravitational units.

Hint: The *K.E.* of the train has to be used up in moving through half a mile against this force. Equate the *K.E.* with the work done :— ($K.E. = F. \times S.$)

152. A man catches a cricket ball of 6 oz. travelling at a speed of 40 ft./sec. and in doing so draws his hand back through one foot. Find the force (in absolute units) exerted by the ball on the hand while coming to rest.

153. A bullet, weighing 50 gm. leaves the muzzle of a gun with a velocity of 500 m./sec. If the barrel is 100 cm. long, find the mean pressure in dynes exerted by the powder upon the bullet.

154. A bullet weighing 25 gm. leaves the muzzle of a gun with a velocity of 500m./sec. It is stopped by an impact against a bone in which it penetrates through a distance of 2·5 cm. Find the force with which the bullet enters the bone.

155. A body moving horizontally with a velocity of 10 ft./sec. meets a smooth inclind plane rising 1 in 2. How high up the plane will it go in virtue of its motion ?

If m be the mass of the body and h the height to which it rises against gravity, the *P.E.* acquired by the body will be $m \times g \times h$. Also its *K.E.* at the foot of the incline $= \frac{1}{2}m \times 10^2$

Since all this kinetic energy is expected to be converted into potential energy of the body, as there is no waste due to friction etc, we have

$$m \times g \times h = \tfrac{1}{2}m \times 10^2$$
$$\therefore \quad h = 1\tfrac{9}{16} \text{ ft.}$$

156. A cricket ball, weighing 6 oz., falls from a height

of 64 ft. vertically downwards. What is its kinetic energy when it reaches the ground ?

157. A stone is dropped from the top of a tower 20m. high. If the weight of the stone is 0·5 kg,, find the kinetic and potential energies (in joules) it possesses at the end of the second second.

158. A couple of movement 200 lbs. ft. acting on a see-saw rotates it through (1) an angle of 5°, (2) an angle of 30°. Determine the work spent in each case.

The work done is the couple multiplied by the angle (in radians) turned through, and

$$5^\circ = 0{\cdot}0873, \quad 30^\circ = 0{\cdot}5236 \text{ rad.}$$

Hence (1) $\text{work} = \left(\dfrac{200 \times 5 \times \pi}{180} = \right) 17{\cdot}46$ ft. lbs.

and (2) $\text{work} = (200 \times 0{\cdot}5236 =)\ 104{\cdot}72$ ft. lbs.

7. Machines

159. Find the length of the power-arm of a lever to raise a 200 lb. wt. kept at a distance of 3 ft. from the fulcrum, if the power applied is 50 lb. wt. State the mechanical advantage of the machine.

160. A pair of nut crackers is 20 cm. long and a nut is placed 5 cm. from the hinge. If the force required to crack the nut be 250 gm. wt., fiind the force which would crack the nut directly.

161. The oar of a boat is 10 ft, long and the row-lock 2 ft. from the handle. If the oarsman pulls with a force fo 100 lb , determine the force exerted in propelling the boat.

162. A bucket of water, weighing 50 lb. is lifted by a wheel and axle. If the axle is 3 in. in radius and the wheel 1·5 ft., find the least effort applied to lift the bucket.

Here the load moves through a distance of $(2\pi \times 0{\cdot}25 =)$ $0{\cdot}5\pi$ ft. during the time the effort moves through $(2\pi = 1{\cdot}5 =)3\,\pi$ft. Hence the work done on the machine $F \times S = (P \times 3\pi =)3\pi P$ ft. lb. where P is the power in lb. wt., while the work got from the machine

$$=(50 \times 0{\cdot}5\pi =)25\pi \text{ ft. lb.}$$

As the fraction is to be neglected, both the amounts of work are equal. $\therefore\ 3\pi P = 25\pi$, $\therefore\ P = 8\frac{1}{3}$ lb. wt.

163. In the second system of pulleys, there are 3 pulleys in each block. Find the effort required to raise a load of 100 lb., if the weight of the lower block is 8 lb. and the string is tied to the upper block. Neglect fraction.

As the string is tied to the upper block and as there are 3 pulleys in each block, the number of stands between which the weight is divided is 6. Hence when the effort moves through 6 units of distance, the load moves through one unit.

But energy supplied = energy got from the machine.

$\therefore$ effort $\times 6 =$ load $\times 1$, because fraction is neglected.

$$\text{Hence effort} = \frac{(100+8) \times 1}{6} = 18 \text{ lb. wt.}$$

164. In the second system of pulleys there are 4 pulleys in each block. Find the average force of friction in each pulley, if 69 lb. can be raised by a force of 10 lb. wt. The weight of the lower block is 10 lb. and the string is tied to the upper block.

165. Find the weight of the lower block in the second system of pulleys, if a load of one ton can be raised by a force of 226 lb. wt. Each block contains 5 pulleys and string is tied to the upper block. Neglect the friction.

166. In the second system of pulleys, an effort of 70 lb. wt. just supports a load of 200 lb. wt., and an effort

of 52 lb. wt. supports a load of 110 lb. wt. What is the weight of lower block and the force ratio ? The friction may be neglected.

Hint : Assume w the weight of the lower block. Since the force ratio in the first case is equal to the force ratio in the second case, calculate w and hence the ratio of the total load raised to the effort applied.

167. A man wants to load a safe, weighing 1000 lbs, in a car which is 3 ft. high from the ground. How long an inclined plank should be used, if he is able to exert a maximum forec of 150 lb. wt. ? Friction is negligible.

Let l ft. be the length of the plank required, then, energy that can be supplied by the man $= l \times 150$ ft. lb. The work to be done by the 'machine'

$$=(3 \times 1000 =)\ 3000 \text{ ft. lb.}$$

Hence neglecting friction, $150\,l = 3000$, $\therefore l = 20$ ft.

168. A cooly wants to raise a bale of cloth, weighing 480 lb., upto a ware-house door which is 30 in. above the ground. How long an inclined plane must he use, if he can exert a force of 120 lb. wt.

169. Find the horizontal force to support a weight of 100 lb. on a smooth inclined plane having a gradient of 5 in 13. What force will be required in a direction parallel to the plane ?

170. A screw has 10 threads to an inch. Find the value of a greatest weight that can be raised by applying an effort of 100 lb. wt. at the end of a lever 2 ft. long. Neglect friction.

The work done in one turn by the effort applied

$=$ effort $\times$ the distance through which the point of application moves

$= (100 \times 2\pi \times 2 =) 400\pi$ ft. lb.

The work done by the screw during one turn
$=$load $\times$ height through which it is raised.

$$= \left(W \times \frac{1}{10\times 12}\right) \tfrac{1}{120} W \text{ ft. lb.}$$

where W is the weight in pounds that can be raised.

$\therefore \quad \frac{1}{120} W = 400\pi$

$W = 150816$ lb. wt. *i.e.* 67·43 tons.

171. A man is able to create a pressure of 6284 lb.wt. on the press-board of screw press by applying a force. of 50 lb. wt. at the end of the lever 2 ft. long. Find the pitch of the screw.

172. A body weighed successively from the two arms of a balance has apparent weights 24 lb. and 25 lb. respectively. Find the correct weight of the body and the ratio of the arms of the balance.

Let a and b be the arms of the balance, W the correct weight of the body; then taking the moments of the force about the fulcrum, we have

$W \times a = 24b$ and $W \times b = 25\,a$.

$\therefore \quad W^2 = 24 \times 25, \qquad \therefore \quad W = (\sqrt{24\times 25} =)$ 24·49 lb.

and $\quad a/b = \sqrt{24/25}.$

173. The arms of a false balance are in the ratio of 20 : 21. What will be the loss or gain to a customer, who wants to buy 5 lb. of goods at 8 as. per lb., if the tradesman places the articles to be weighed at the end of the longer arm ?

Hint : Taking the moments of the forces acting at the ends of the arms, determine the actual weight of the goods the customer gets. Hence calculate the loss or gain.

LIGHT

1. Velocity of light

Light travels through space with a velocity of 186000 mi./sec. or 299800 km./sec.

1. Earth's orbit is 186×10^6 mi. in diameter. Find the time required for the light to travel from the sun to the Earth. *Assume the orbit to be circular.*

2. Find the distance of the star Arcturus, if light from it requires $3{\cdot}2 \times 10^3$ sec. to come to the Earth.

3. If the star Sirius, distant 52×10^{12} mi. from the Earth, were to explode suddenly, how long would it be before the astronomers on coour planet uld detect the fact?

2. Pinhole-Camera

4. A pinhole is made in the shutter of a dark room, and a screen is held at a distance of 6 ft. from the shutter. Find the size of the image of a tree which is 30 ft. high and is at a distance of 40 ft. from the shutter.

Since the light travels in a straight line, from the properties of similar triangles, we find that

$$\frac{\text{size of the object}}{\text{distance of the object}} = \frac{\text{size of the image}}{\text{distance of the image}}$$

$$\therefore \frac{30}{40} = \frac{I}{0} \qquad \therefore \text{ size of the image} = 4{\cdot}5 \text{ ft.}$$

5. A pinhole-camera is 10 in. long. An image of a building 50 ft. high is obtained on the plate. If the image measures 4 in. in height, determine the distance of the building from the pin-hole.

6. Outside a small hole in a wall of a dark room 15ft. square, is a tower 24 ft. high and 60 ft. away from the

wall. The image of the tower falls on the other wall of the room. What is the size of the image ?

Hint : As the room is 15 ft. by 15 ft. in base-area. the distance of the image from the hole in the wall is 15 ft.

3. Photometry

Suppose

I = the intensity of illumination on a surface,

L = the illuminating power of a source of light,

d = the distance of the source from the surface,

θ = the angle of incidence of the rays on surface,

then

(*i*) $I \propto 1/d^2$, L remaining constant.

(*ii*) $L \propto d^2$, I ,, ,,

(*iii*) $I \propto L$, d ,, ,,

(*iv*) $I \propto \cos\theta\, L$, d ,, ,,

$$\text{Foot-Candle}^* = \frac{\text{candle-power}}{\text{foot}^2}$$

7. Compare the intensity of illumination on a surface due to a lamp of 100 c. p. (candle-power) placed 20 ft. away from the surface to that due to a similar lamp placed 50 ft. away from the same surface.

Applying the Law of Inverse Squares, as L is constat

$$\frac{I_1}{I_2} = \frac{d_2{}^2}{d_1{}^2} = \frac{50 \times 50}{20 \times 20} \text{ or } \frac{25}{4}.$$

---

*'**Foot-candle**' is a unit of intensity of illumination. It is equal to the intensity of illumination received by a surface one foot away from a standard candle.

The other unit for the same is '**Lux**'. It is the illumination produced by a source of unit luminosity on a screen situated one metre away from the source. The **Lux** may be called a **metre-candle**.

'**A Standard-Candle**' is defined as a candle of pure sperm wax $\frac{7}{8}$ in. in diameter and weighing six to the pound, and burning at the rate of 120 grains an hour.

8. Two lamps are placed at distances of 30 cm. and 40 cm. respectively from the screen of a Bunsen's Photometer. If the brightness of the spot is the same on both the sides, find the ratio of the illuminating powers.

When the two lamps are balanced, intensity of illumination due to both the lamps is the same. Hence,

$$\frac{L_1}{L_2}=\frac{d_1^2}{d_2^2} \therefore \frac{L_1}{L_2}=\frac{900}{1600} \text{ i.e., } \frac{9}{16}.$$

9. How many candles are required to produce, at a distance of 60 cm., the same illumination as one candle at a distance of 10 cm. from a screen ?

9*a*. A lamp of 30 c.p. was placed at distance of 60 c.m. from a grease spot and another lamp of 10 c.p. was kept at a distance of 60 cm. from the same screen on the other side of the spot. Find the position of a third lamp of 4 c.p. which would help one of the above lamps so as to have the spot equally illuminated.

10. Compare the intensity of illumination on the floor of a hall due to an incandescent lamp of 100 c.p. kept at a height of 50 ft. with that due to an arc lamp. of 1000 c.p. suspended at a hight of 100 ft.

Here $I_1 \propto \dfrac{100}{2500}$ and $I_2 \propto \dfrac{1000}{10000}$ $\therefore L \quad \propto \quad O.P.$

$$\frac{I_1}{I_2}=\frac{100}{2500}\times\frac{10000}{1000}=\frac{2}{5}.$$

11. The candle powers of two lamps are to each other as 1 : 2, and their distances from a screen are as 4 : 5. What are their relative intensities of illumination on the screen?

12. The distance between a 90 c.p. lamp and a 10 c.p. lamp is 20 ft. Where must a screen be placed to get it equally illuminated by both the lamps ?

Let x be the distance (in feet) of the screen from the 10 c.p lamp. Then its distance from the 90 c.p.

lamp will be $(20-x)$ ft.

When the screen is equally illuminated by each lamp, we have,

$$\frac{10}{x^2}=\frac{90}{(20-x)^2}\therefore 20-x=\pm 3x. \text{ Hence } x=5 \text{ ft. } or -10 \text{ ft.}$$

There are two positions of the screen corresponding to the two values of x. One is 5 ft. from 10 c.p. lamp between the two lamps and the other 10 ft. to the left of the 10 c.p. lamp, *i.e.* 30 ft. from the 90 c.p. lamp.

Note :— The first value ($x=5$ ft.) gives the arrangement for a Bunsen's photometer while the second one ($x=-10$ ft.) gives the arrangement for a Rumford's photometer.

12*a*. The distance between two lamps of 8 and 16 candle-powers is 6 ft: Show that there are two positions on the line joining the lamps in which a screen may be placed so as to receive equal illuminations from each of the lamps and determine these positions ?

13. A candle sends it rays against a vertical plane screen. If the candle is removed to thrice the original distance and the screen is so placed as to make an angle of 60° with the original position, what is the ratio of the intensities of light in two cases ?

If I'_2 be the intensity of illumination in the second case when the surface A is as in the first case and only the distance is changed, then taking I_1 as the intensity of illumination in the first arrangement, we get $I_1/I_2'=9d^2/d^2=9$. The surface makes an angle of 60° to the first position, hence the intensity of illumination decreases. Let this be I_2, then:

$$\frac{I'_2}{I_2}=\frac{A}{A\cos 60}=\frac{2}{1}\therefore\frac{I_1}{I_2}=\frac{I_1}{I_2'}\times\frac{I_2'}{I_2}=9\times 2=18.$$

***14** A gas flame when burning at the rate of 5 cu. ft./hr. placed at a distance of 100 in. from the screen illuminates

it equally with a candle placed at 30 in. burning at the rate of 1 oz. in 4 hrs. The gas costs Rs, 3/–per 1000 cu.ft., and the candle Re. 1/- per lb. Compare the cost of lighting a room with gas and candle respectively.

[B. A. 1924]

Hint: Compare the illuminating powers, then determine the number of candles equivalent to one lamp; hence calculate the cost per hour of the quantities of the gas and the candle consumed from the prices mentioned.

15*. A lamp when kept at a distance of 45 cm. from a grease spot balances another lamp at a distance of 51 cm. from the other surface of the spot. A glass plate, which absorbs 19% of the light incident on it, is interposed between the stronger lamp and the spot. By what distance must this lamp be taken nearer the spot to illuminate it again equally ?

16. 200 c.p. electric lamps are used for street lighting in a big city. The minimum illumination at any point along the street is desired to be the same as is obtained with an ordinary 4 c.p. lamp at a distance of 33 ft. Find the number of lamps that would be placed along the street 2 m. long.

Hint: The total minimum intensity of illumination, due to any two successive lamps, at a point on the road midway between the two, must be as much as that given by a single lamp of 4 c.p. at a distance of 33 ft. Hence find the number of lamps. Add one more to this number.

***17.** A big hall $40' \times 60'$ is to be illuminated by two lamps placed symmetrically along its length so as to provide for minimum illumination at any point within it as is provided by an ordinary 4 c. p. table lamp at a distance of 2·5 ft. Find the minimum candle power of the gas-lights for the purpose.

Hint: Lamps when symmetrically arranged will have the positions 20 ft, away from both the side-walls, so that the distance between the two will be also 20 ft.

The illumination due to both the lamps will be minimum at the corners of the hall when they are properly arranged. The intensity of illumination to be compared with is given by $4/2{\cdot}5^2$.

18. What is the candle-power of a lamp which produces an illumination of 10 foot-candles on a surface at a distance of 5 ft. ?

Since foot-candle $= \dfrac{\text{candle-power}}{\text{foot}^2}$, c.p. $= (\ 10 \times 5^2 =\ )\ 250.$

19. If 4 ft-candles is the proper illumination for reading, how far from the page of a book would you keep a lamp of 16 c.p. ?

20. 25 sec. is the proper time for exposure, when printing a photograph by a gas-light kept 10 in. away from the printing frame. Determine the time of exposure required in printing from the same negative, when the same light is keep 20 in. away.

Hint: The time of exposure varies inversely as the intensity of illumination.

21. A photographer finds that a print exposed for 5 sec. by keeping at a distance of 4 ft, from an electric lamp is over-exposed. He desires, therefore, to give $\frac{4}{9}$ of the exposure to another piece of the same paper keeping it exposed to light for the same time as before. Find the distance at which he should keep the printing frame from the lamp.

4. Plane Mirrors

(i) If a mirror is rotated through an angle θ, the

reflected ray is rotated through an angle 2θ, the incident ray having the same path both before and after the rotation of the mirror.

(ii) In the lamp and scale arrangement, if D is the distance of the scale from the mirror and d the distance through which the spot of light gets shifted of the scale when the mirror is deflected through α radians, then we have $\tan 2\alpha = d/D$. If α is very small, $2\alpha = d/D$.

22. Find the true altitude of a star, when it is found to be 60° by means of an artificial horizon. The mirror employed for the artificial horizon is inclined at an angle of 5° to the horizontal and slopes down from the observer.

23. A scale was kept 50 cm. away from the mirror of a mirror galvanometer and a spot of light was found to move through a distance of 20 mm., when a current was passed through the galvanometer. Find the angle of deflection of the mirror.

Since the angle is very small, $\tan 2\alpha = 2\alpha$ From

the relatian, $\tan 2\alpha = \dfrac{d}{2D}$, we get

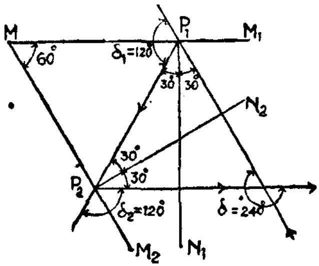
$$\alpha = \frac{d}{2D} = \frac{2}{2 \times 50} = 0{\cdot}02 \text{ radian.}$$

24. Calculate the angle of deflection when a spot of light moves through (1) 100 cm. (2) 12·5 mm., the mirror and the scale being at the usual distance of 100 cm.

[I.Sc. 1925]

***25**. A ray of light is incident on a plane reflecting surface at an angle of 30°, find the deviation produced in the incident ray. What will be the deviation, if the ray suffers a reflection again at a surface inclined at 60° to the first one ?

(i) The deviation δ_1 is $\pi - (i + r) = \pi - 2i$
$= 180° - 2 \times 3^{\circ\circ} = {}^{1\circ\circ\circ}$.



Deviation (i) 120°
Deviation (ii) 240°

(ii) The deviation δ_2 due to the other surface is

$$\pi - (i_1 + r_1) = \pi - 2i_1$$

but if θ be the angle between the two surfaces, $\theta = i + i_1$.

$\therefore$ the deviation δ due to the successive reflections at _oth _h_ _ s

$= 2\pi - 2\theta = 360° - 120° = 240°$

5. Spherical Mirrors

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f} \text{ in case of mirrors}$$

$$\text{Magnification} = \frac{\text{image } (I)}{\text{object } (O)}, \text{ (for linear magnification).}$$

$$\text{Magnification} = \frac{\text{linear size of the image}}{\text{linear size of the object}}$$

$$= \frac{\text{the distance of the image* } (v)}{\text{the distance of the object } (u)}$$

26. The focal length of a concave mirror is 2 m. What is the position of the image, when the object is at a distance of (*a*) 1 m., (*b*) 2 m., (*c*) 3 m,, (*d*) 4. m. and (*e*) 5 m. respectively from the mirror ?

From the general formula for mirrors, we have

---

* All the distances measured from a mirror (or a lens) in an opposite sense to that in which the incident light falls upon the mirror (or the lens) are taken as positive (+ve) while those in the same sense as the incident light as negative (-ve). *To avoid confusion we restrict to the usual convention of signs. Refer author's F. Y. Practical Physics (P. U. C.) Appendix II.*

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \quad \therefore \quad v = \frac{f\,u}{u-f}$$

$\therefore$ (*a*) $v_1 = \frac{2\times 1}{1-2} = -2$ m. *i.e.* the image is 2 m. behind the mirror.

(*b*) $v_2 = \frac{2\times 2}{2-2} = \infty$. The image is at infinity.

(*c*) $v_3 = \frac{2\times 3}{3-2} = 6$ m. *i.e.* the image is 6 m. in front of the mirror.

(*d*) $v_4 = \frac{2\times 4}{4-2} = 4$ m. The image is 4 m. in front ,,

(*e*) $v_5 = \frac{2\times 5}{5-2} = 3\frac{1}{3}$ m. The image is $3\frac{1}{3}$ m. ,, ,,

27. Determine the position and the size of the image of an object 10 in. high placed 30 in. away from a concave mirror of 20 in. radius of curvature.

28. An object 4 cm. high is placed in front of a concave mirror and its image is received on a screen placed at a distance of 15 cm. from the object. If the image is 5 cm. high and parallel to the object, find the focal length of the mirror.

Here $v = u + 15$ and $v/u = 5/4$ $\therefore$ $u = 60$ cm. and $v = 75$ cm. $\therefore$ from the usual relation for mirrors we get, $\frac{1}{f} = \frac{1}{75} + \frac{1}{60}$ $\therefore$ $f = (\frac{100}{3} \doteq)$ 33·3 cm.

29. An image formed by a concave mirror is 4 times the object in size. The focal length of the mirror is 20 in. Determine the position of the object from the mirror.

Hint : There are two positions of the object; one giving a real image in front of and the other giving a virtual image behind the mirror. Take the proper algebraic sign for the magnifications.

* **30.** Find the size and the nature of the image of an object which measures 4 in. by 3 in. when it is held at a distance of 30 in. in front of a concave mirror of focal length 20 in.

Hint: Find the length of each side in the image.

31. If an image produced by a convex mirror in one-half of the object in size, find the distance of the object from the mirror. The focal length of the mirror is 30 cm.

Focal length f and the distance of the image v are negative quantities in the case of a convex mirror. Hence we have $v/u = -1/2 \quad \therefore \quad 2v = -u$. Also

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{-30} \quad \therefore \quad \frac{2}{-u} + \frac{1}{u} = \frac{1}{-30} \quad \therefore \quad u = 30 \text{ cm.}$$

32. An object 2 cm. in length is held at a distance of 50 cm. in front of a convex mirror ($r = 30$ cm.). Find the nature, position and the magnitude of the image.

32a. An object is (*a*) 40 cm. (*b*) 20 cm. (*c*) 5 cm. in front of a convex mirror of 20 cm. radius of curvatur . Where is the image in each case, and what is th magnification ? [I.Sc. 1935

33. A plane mirror is placed 22·5 cm. in front of a convex mirror, the top of the plane mirror being on a level with the centre of the convex mirror. A rod is placed 50 cm. in front of the convex mirror, and its images in the two mirrors coincide without parallax. Find the focal length of the convex mirror. [I.Sc. 1924]

Hint: The distance of the object from the convex mirror be measured directly. That of the image from it is given by the difference between the distance of the object from the plane mirror and that between the two mirrors.

34. A plane mirror and a convex mirror are placed

facing each other at a distance of 26 cm., the upper edge of the plane mirror being on a level with the centre of the surface of the convex mirror. When a luminous vertical slit is placed behind the plane mirror at a distance of 60 cm. from the convex mirror, an image of the slit is seen in the plane mirror coinciding with the slit itself without parallax. Determine the focal length of the convex mirror.

Hint : The image formed by the convex mirror gives its own image by reflection in the plane mirror. This latter is formed in the position of the slit and its distance behind the mirror being known, the original image in the convex mirror must be at an equal distance in front of the plane mirror. Thus find v for the convex mirror, u is given.

35. A concave mirror (r = 30 cm.) and a convex mirror (r = 20 cm.) areplaced facing each other and 50 cm. apart. An object 5 cm. long is placed perpendicular to the common axis of the mirrors in front of the concave mirror at a distance of 24 cm. from it. What will be the position and the character of the image formed by the two reflections, first at the concave and then at the convex surface ?

6. Refraction at a plane surface

Refractive index μ of a substance with respect to air is given by

$$\text{(i)}\quad \mu = \frac{\text{velocity of light in air*}}{\text{velocity of light in the substance}} = \frac{v \text{ air}}{v \text{ substance}}$$

$$\text{(ii)}\quad \mu = \frac{\text{sine of angle of incidence } i}{\text{sine of angle of refraction } r} = \frac{\sin i}{\sin r}.$$

* To be more precise, the velocity of light in vacuum should be considered. However for ordinary purposes velocity of light in air is taken.

(iii) $\mu = \dfrac{\text{real thickness of the substance}}{\text{apparent thickness of it}} = \dfrac{H}{h}$

(iv) $\mu = \dfrac{1}{\text{Sin } C}$, where C is the critical angle for the surface separating the two given media. It may be defined as the largest angle of incidence which allows any of the incident light to pass through into the rarer medium.

If a ray of light passes from one medium A to the other B and again emerges in A then,

(v) ${}_A\mu_B \times {}_B\mu_A = 1$ or ${}_A\mu_B = 1/{}_B\mu_A$

(vi) If the refraction takes place through a pile of a number of parallel plates of media $A, B, C, D, \ldots A$ say, then

$$ {}_A\mu_B \times {}_B\mu_C \times {}_C\mu_D \times {}_D\mu \ldots \mu_A = 1 $$

(vii) $\mu = \sin\frac{1}{2}(\theta + \delta_m)/\sin\frac{1}{2}\theta$ for a prism. If θ be very small, we can write it as $\theta\mu = \theta + \delta_m$ *or* $\delta_m = \theta(\mu - 1)$ Hence

(viii) dispersion between v and r lights (say)

$$ \delta_v - \delta_r = (\mu_v - \mu_r) \theta $$

36. Find the refractive index of water if the velocity of light in it is 143330 mi./sec. and in vacuum is 186330 mi./sec.

37. The angle of incidence of a ray of light on a plane glass surface is 50° and the corresponding angle of refraction is 30°. Find the refractive index of the glass.

We have, $\mu = \dfrac{\sin i}{\sin r}$ and $i = 50°$, $r = 30°$.

$$ \therefore \quad \mu = \frac{\sin 50°}{\sin 30°} = \frac{0{\cdot}766}{0{\cdot}500} = 1{\cdot}532. $$

38. A ray of light is incident on a plane surface of a glass cube at an angle of 60° to the normal and the refractive index of glass is 1·5. What is the angle of refraction ?

39. Find the angle of refraction of a ray of light

which passes from air into water at an angle of 65°, ($_a\mu_w = \frac{4}{3}$).

40. Find the deviation in the path of a ray of light which is incident on a plane refracting surface of a glass cube at an angle of 40° (μ for glass is 1·5).

Hint: The angular displacement of the incident ray on refraction is called *deviation*. Find the angle of refraction, thence the deviation.

41. A thick glass slab appears to be 10 cm. thick, when viewed at right angles to its surface. Determine the refractive index of glass, if its true thickness is 15 cm.

$$\mu = \frac{\text{real thickness of the slab}}{\text{apparent thickness of it}} \quad \therefore \mu = \frac{15}{10} = 1{\cdot}5.$$

42. How much does a picture fixed to the bottom of a glass-cube appear raised to a perpendicular vision? The cube is 6 in. thick and the refractive index of its material 1·5.

43. When a microscope is focussed upon a pencil mark on a piece of paper the reading on the scale is 14.39 cm. A glass block is then placed on the paper and the microscope is again focussed on the mark, the reading is 18·43 cm. When focussed on the upper surface of the block, the scale reading is 26·51 cm. Determine the refractive index of the material of the slab.

Hint; Find the actual depth and the apparent depth of the mark on the paper below the upper surface of the block. Hence determine μ.

44. The critical angle for glass is 41°. What is the index of refraction?

***45.** A small electric lamp is placed at the bottom of a large dish containing a liquid of refractive index $\frac{5}{3}$, the depth of the liquid over the lamp being 10 *cm*. Find the

position and the radius of the smallest circle of card which when placed on the liquid, entirely prevents the lamp being seen. [I.Sc. 1929]

Hint: Assuming that the image lies on the vertical passing through the lamp, the maximum angle of incidence on the surface for emergence outside should exceed the critical angle *just a little* to cut off the light from being seen. μ is given, hence determine the radius using trigonometrical relations.

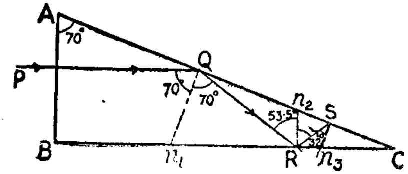
46. Find the depth of a transparent liquid kept in a trough, a luminous object at the bottom of which cannot be seen outside the boundary of a circle of radius 4 cm., and whose centre is in the surface of the liquid and above the point. (${}_a\mu_l = 1{\cdot}25$).

47. A cube of a side 1 in. is constructed of a transparent material of refractive index 1·65. Calculate the smallest radius of circular discs of card required to be placed centrally over each face of the cube so that a small air bubble at the centre shall be invisible from an external point.

48. Find the minimum refractive index of a substance required to turn a ray of light through 90° by using a right-angled prism. Critical angle of the material of the prism is 45°.

49. The angles of a glass prism are 90°, 70° and 20°. A ray of light enters the prism normally at the face bounded by the angles 90° and 70°. If the critical angle for glass is 41°, show by drawing a neat figure that the ray will suffer two internal reflections before it emerges from the prism. Mark all the angles accurately. [B.A. 1927]

PQRS is a ray suffering one reflection at Q on AC ($i_1 = 70°$) and other t R on BC ($i_2 = 50°$). Finally it is incident on face AC at S at an angle of 30° which is less than 41°, so it emerges at S.



(Thus the ray suffers two internal reflections.)

50. The refractive index for water is $\frac{4}{3}$, and for glass $\frac{3}{2}$. A ray of light travelling in water is incident at an angle of 30° upon a plane water-glass interface. What is the angle of refraction of the ray ?

Hint: Calculate the value of ${}_w\mu_g$. Angle of incidence is given. Hence find the angle of refraction.

51. What is the critical angle between water and carbon disulphide, if the refractive index ${}_a\mu_w = 1{\cdot}33$ and ${}_a\mu_c = 1{\cdot}63$?

52. Calculate the refractive index of a prism whose angle is 60° and the minimum deviation obtained for homogeneous light is 51·5°. What is the critical angle for light travelling in such a medium ?

53. The following observations were noted to determine the refractive index of a prism using a spectrometer.

Angle subtended between the two positions of the telescope for viewing a ray of light reflected from the two faces of the prism was 120·04°.

Direct reading of the telescope when focussed on the slit of the collimator = 14·7°

Reading when telescope was adjusted for

(i) the minimum deviation on one side = 54·9°

(ii) the minimum deviation on the other = 334·7°

Calculate the refractive index.

54. A prism is to be made of flint glass of which the refractive index for the mean ray is 1·587. To what angle must it be ground to produce a minimum deviation of 1·7° ?

As the minimum deviation $\delta = \theta(\mu - 1)$, where θ is the angle of the prism and μ the refractive index of its material, we get $1{\cdot}7° = \theta\ (1{\cdot}587 - 1)\ \therefore\ \theta = 2{\cdot}896°$.

55. The refractive index of rock-salt is 1·54. Find the deviation produced by a rock-salt prism of the angle 1°. Determine the angle of a prism of the same substance to produce a deviation of 54′.

*56. The dispersion produced by an equilateral prism of flint glass is 2·16°, and that produced by a similar prism of crown glass is 1·38°. What should be the angle of a crown glass prism which can render a flint glass prism of 45° achromatic ? [B.A. 1926]

The dispersion is given by $\delta_v - \delta_r = (\mu_v - \mu_r)\,\theta$

$\therefore$ for the flint glass prism $(\mu_v - \mu_r)\ 60 = 2{\cdot}16$

$$\therefore\ (\mu_v - \mu_r) = 2{\cdot}16/60$$

Similarly for the crown glass prism $(\mu'_v - \mu'_r) = 1{\cdot}38/60$. For achromatism, $(\mu_v - \mu_r)\theta_t = (\mu'_v - \mu'_r)\,\theta_c$

$$\therefore\ \theta_c = \frac{2{\cdot}16}{60} \times \frac{60}{1{\cdot}38} \times 45 = 70{\cdot}42°.$$

7. Refraction at a curved surface

For a single spherical surface of refractive index μ and the radius of curvature R, when an object is (i) in a rarer medium; (ii) in a denser medium

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R} \qquad\qquad \frac{1}{v} - \frac{\mu}{u} = \frac{1 - \mu}{R}$$

*The side on which light is incident is called the front of the surface. All distances measured in front of the surface are taken as positive and those measured behind the surface are negative. Take the distance from the pole of the surface.

57. A small air bubble is in a sphere of solid glass of 8 cm. radius. It is situated 2 cm. from the centre and is viewed from the side to which it is nearest. Where will it appear to be ? ($_a\mu_g = 1{\cdot}4$).

The actual distance u of the bubble from the surface at which it is viewed is 6 cm. To find the apparent distance v, since the ray starts from the object in the glass and comes out in air we have to use the relation

$$\frac{1}{v} - \frac{\mu}{u} = \frac{1-\mu}{R}$$

$\therefore$ we get $\dfrac{1}{v} = \dfrac{1{\cdot}4}{6} + \dfrac{1-1{\cdot}4}{8} = \dfrac{1}{5{\cdot}45}$ $\quad\therefore\quad v = 5{\cdot}45$ cm.

from the surface of the sphere, on a radius through the object.

58. A small object enclosed in a sphere of solid glass of 6 cm. diameter appears to be 2 cm. from the surface when looked at along a diameter, Calculate the true position of the bubble. ($_g\mu_a = \frac{2}{3}$),

59. A glass sphere is 10 cm. in diameter. A small air bubble inside the sphere appears to be 2 cm. from the nearer surface of the sphere when it is viewed along that line which passes through the bubble and the centre of the sphere. What is the true position of the bubble? $\mu = 1{\cdot}5$.

8. Spherical lenses

* Focal length of a lens is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad (1)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (2)$$

*Note that in the case of lenses *2f is not equal to r,* Also in the application of the formula, remember the convention of algebraic signs for f and v. Refer to the footnote on page 91 and also section 7 above.

$f = -\frac{D^2 - d^2}{4D}$ for a converging lens ; here D is the distance between the object and a screen, while d is the distance between the two positions of the lens at which it gives an image on the screen. (3)

60. An object, 6 cm. high. is placed at a distance of 120 cm. from a thin convex lens and an image is formed on the other side 40 cm. away from the lens. Find the focal length of the lens. Calculate the magnification also. What is the size of the image ?

Since $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for a lens, we have $\frac{1}{f} = \frac{1}{-40} - \frac{1}{120}$

where $v = -40$ cm. $u = 120$ cm.

$$\therefore \quad \frac{1}{f} = -\frac{3+1}{120} \text{ or } f = -30 \text{ cm.}$$

Again the magnification $\frac{v}{u} = \frac{40}{120} \quad \therefore \quad \frac{I}{O} = \frac{40}{120} = 0{\cdot}33.$

Hence I the size of the image $= (6 \times \frac{40}{120} =)$ 2 cm.

61. Find the minimum length of a photographic plate required to take a full size image of a man 5 ft. – 6 in. high standing at a distance of 20 ft. from the camera-lens of 10 in. focal length.

Hint : Find v using the values of u and f. Then determine the magnification. Size of the object is given, hence determine the size of the image. That gives the required length of the plate.

62. A lantern slide, 8 cm. square, is to be projected on a screen 6 m. away from the lens so as to give a picture 3 m. square. Find the focal length of the lens required.

Hint : Take the linear magnification. Refer to Ex. 30.

63. Where must an object be placed in front of a convex lens (f = 20 cm.) to form an image 4 times as large

as the object (a) when the image is real, (b) when it is virtual ?

(a) $\frac{I}{O} = -4 \quad \therefore \frac{v}{u} = -4$, as the image is on the other side of the lens when it is real. $\therefore v = -4u$. Again from the general formula for lenses, after putting in the value of v and f, we have , $-\frac{1}{4u} - \frac{1}{u} = -\frac{1}{20}$, hence $\frac{5}{4u} = \frac{1}{20}$

$$\therefore \quad u = 25 \text{ cm}.$$

(b) $\frac{I}{O} = \frac{v}{u} = 4$, as the image is on the same side of the lens as the object v is positive. $\therefore v = 4u$, therefore as in (a) using the general relation, we get

$$\frac{1}{4u} - \frac{1}{u} = -\frac{1}{20} \text{ hence } \frac{3}{4u} = \frac{1}{20} \qquad \therefore \quad u = 15 \text{ cm}.$$

64. A magnifying glass is held 5 cm. in front of a book and the print appears 3 times as large. What is the focal length of the lens ?

Hint : Here values of u and the linear magnification are given. Hence calculate f. The image is virtual.

65. An object is 6 ft. from a wall ; the two positions at which a given convex lens may be placed to get a sharp image of the object on the wall are 2 ft. apart. Calculate the focal length of the lens.

***66.** A convex-lens is placed in front of an object so that the image is of the same size as the object. The lens is then moved towards the object till the image is twice the size of the object. Find the focal length of the lens, if the distance through which the lens is moved be 10 cm.

[I.Sc. 1916]

In the first arrangement the magnification m_1 is unity, hence, $-v = u$. In the second arrangement the

magnification $m_1 = -v' / (u - 10) = 2 \quad \therefore \quad v' = 2u - 20$ and $u' = u - 10$. Taking the usual formula for lenses we get

$$\frac{1}{f} = \frac{1}{v'} - \frac{1}{u'} = -\left(\frac{1}{2u - 10} + \frac{1}{u - 10}\right) = -\frac{1+2}{2u - 20}$$

$$\therefore f = -(2u - 20)/3 \text{ ; but we have also } u = -v = -2f.$$

$$\therefore 3f = 4f + 20 \text{ or } f = -20 \text{ cm.}$$

Note : v' is not equal to $v+10$. The distance between the object and the screen has to be increased because the value of $u+v$ is minimum and equal to $4f$ when the magnification is unity.

67. A convex lens is placed in front of a plane mirror so that the lower edge of the mirror is on a level with the upper edge of the lens. An object is put between the two along the principal axis of the lens and perpendicular to it. When the distance between the lens and the mirror is 30 cm. and that between the object and the lens is 10 cm., the images due to both the lens and the mirror coincide without parallax. Calculate the focal length of the lens.

68. An object 9 in. high is placed at a distance of 45 cm. from a convex lens of 20 cm. focal length A plane mirror is placed at a distance of 40 cm. from the lens on the other side. Find the position, the nature and the size of the image formed.

Hint : Find the distance v_1 of the image due to the lens, the image of this by the plane mirror is an object for the lens for the final image. Hence the final image is erect. Find the required distance using general relations.

69. A beam of light diverges from a point S on the axis of a convex lens ($f = 12$ cm.) and after passing through the lens is allowed to reflect from the surface of a convex mirror. The reflected beam is brought to a focus by the lens at a point coinciding with S Find the focal length

of the convex mirror, if the distance between the lens and the mirror is 10 cm. and that between the source and the mirror is 30 cm.

Hint: The incident rays are reflected back along the same path of incidence by the mirror; therefore the position of the first image due to the lens is the centre of curvature of the mirror. First find this position.

70. Calculate the focal length of a lens from the following data :—

Distance between any two outer legs of a spherometer	= 3·9 cm.
Height of the central leg above a plane surface when adjusted on any of the faces of the lens	= 0·168 cm.
Refractive index of the material of the lens	= 1·48.

Hint: The radius of curvature r_1 of the face on which the light is incident is to be taken negative while r_2 the radius of the other face positive. (Why ?)

71. Calculate the refractive index of water if a hollow converging lens filled with it has a focal length of 36 cm., the radii of curvature of the surface being 20 cm. and 30 cm. respectively.

72. A biconvex lens of glass of refractive index 1·5 projects a real image of a luminous object situated 120 cm. away from the lens. The radii of curvature were found to be 20 cm. and 40 cm. How far from the object must the screen be placed to receive a clear image of the object ?

[B.A. 1931]

7 A plane-convex lens has focal length 20 cm. and refract e index 1·5. Find the radius of curvature of the curved surface of the lens. [I.Sc. 1929]

Hint: Here $f = -20$ cm., while $r_1 = \infty$, $\therefore$ $1/r_1 = 0$,

73*a*. The radius of curvature of the convex surface of a convex meniscus lens is 20 cm. and that of the concave surface 30 cm. The refractive index of the glass in 1·52. Calculate the focal length. [B Sc. 1930]

Hint: The lens is converging one. Both r_1 and r_2 have the same sign.

73*b*. If a lens of focal length 20 cm. were immersed in water, what would be its focal length in water? (${}_a\mu_g = 1{\cdot}5$, ${}_a\mu_w = \frac{3}{4}$)

${}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1 \therefore {}_g\mu_w = 1/(\frac{3}{2} \times \frac{3}{4}) = \frac{8}{9} \therefore {}_w\mu_g = \frac{9}{8}$.

Again $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$. If r_1 and r_2 are constant $1/f \propto (\mu - 1) \therefore$ the ratio of the focal length of the lens (f_w) in water to its focal length (f_a) in air is

$f_w/f_a = ({}_a\mu_g - 1) / ({}_w\mu_g - 1)$; hence

$f_w = (1{\cdot}5 - 1) \times 20/ (1{\cdot}125 - 1)$ *i.e.* $f_w =$ 80 cm.

74. An object is situated 30 cm. away from the lens. If a virtual image of it is formed on the same side of the lense as the object at a distance of 20 cm. from the lens, determine the focal length and the nature of the lens.

Hint: $v < u$ and is positive. Value of f will be a positive quantity, showing that the lens is a concave one.

75. An object 10cm. high is placed behind a concave lens ($f = 30$ cm.) at a distance of 20 cm. from it. Determine the position and the size of the image formed by the lens.

76. An image one-fourth in size of the object, is seen through a concave lens when the object is at a distance of 30 cm. from the lens. What is the focal length of the lens?

77. A convex lens forms a real image P of an object O. When a concave lens is interposed between P and the lens at a distance of 30 cm. from P, the position of P is shifted to a distance of 45 cm. from the lens interposed. Determine

the focal length of the concave lens.

Hint: Here the object is virtual and the image real. $u = -30$ cm. and $v = -45$ cm.

•78. A pin P is kept at a distance of 44·9 cm. in front of a concave lens L ($f = 65·31$ cm.). A plane mirror M is then interposed facing the pin between it and the lens, so that the top of the mirror is in level with the centre of the lens. A second pin Q is then held vertical behind the lens at a disiance of 30·4 cm., so that the images of P by M and Q by L coincide without parallax. Determine the positions of the plane mirror.

79. A parallel beam of light incident directly on a concave lens, after passing through the lens, is reflected from a concave mirror ($r = 50$ cm.). It then traverses back so that the rays emerging from the lens are again parallel. The distance between the lens and the mirror is 30 cm. Find the focal length of the lens.

Hint: The centre of curvature of the mirror coincides with the principal focus of the lens. Hence $f = (50 - 30)$cm.

80. An object is put in front of a concave mirror (r = 40 cm.) so that its image coincides with itself. A concave lens of unknown focal length is introduced between the two and the position of the object is changed to get its image coinciding with itself. If the distance between the lens and the mirror be 30 cm. and that between the two positions of the object 20 cm., find the focal length of the concave lens.

Hint: The radius of curvature of the mirror is 40 cm. In the second case, the rays from the object at 30 cm. from the lens pass through it, and are reflected back by the mirror along the same path; and thus the rays are perpendicular to the mirror, *i.e.*, radial, showing that the

image by the lens must be at its centre of curvature. Hence $u=30$, $v=10$; both having the positive sign.

9. Powers of Lenses,—Defects of vision.

The **Power** of a lens may be defined as the amount by which it alters the convergence or divergence of rays incident to the lens. Since the convergence (or the divergence) of a spherical lens depends upon its focal length, the **focal power** of a lens is measured as the reciprocal of its focal length.

The practical unit of a focal power is the **Dioptre.** It is the power of a lens whose focal length is one metre.

The focal power of a converging lens is positive while that of a diverging lens is negative for *scientific purposes.*

Focal power $= -\dfrac{1}{f}$ (for scientific purposes),

or ,, $= -\dfrac{100}{f}$ dioptres,

where the focal length f of a lens is taken in centimetres.

81. A convex lens and a concave lens of 0·025 and –0·02 focal powers respectively are placed along the same axis and just touching one another. Find the focal length of the combination.

82. A convex lens of 4 dioptres and a concave lens of – 2 dioptres are held together coaxially to give a single combined lens. Find the power and the focal length of the combination.

The power of the convex lens $=4$

and that of the concave one $=-2$,

$\therefore$ the resultant power $= (4-2=)+2$ dioptres;

hence the focal length $= (100/2=)-50$ cm.

83. A spherical lens of 5 dioptres when combined with another lens coaxially gives the dioptric power equal

to 4·5. Determine the focal length and the nature of the lens combined.

48. An optician finds that a man's shortest distance of distinct vision is 50 cm. What will be the power of the lenses which will enable him to read comfortably at a distance of 30 cm.

The eye cannot see an object placed nearer than 50 cm. then if f_1 be the focal length of the crystalline lens of the eye and d the distance between this lens and the retina, taking $u=50$ cm. $v=d$, we have from the usual relation,

$\frac{1}{d}-\frac{1}{50}=\frac{1}{f_1}$. If f_2 be the focal length of the auxiliary (spectacle)lens required, then, we will have the object 30 cm. away $\therefore \frac{1}{d}-\frac{1}{30}=\frac{1}{f_1}+\frac{1}{f_2}$ since the two lenses are to be very close. Combining the above two equations $\therefore \frac{1}{50}-\frac{1}{30}=\frac{1}{f_2}$. Hence $\frac{1}{f_2}=-\frac{1}{75}$ or $f_2=-75$ cm.

and the power in dioptres is $-\frac{100}{-75}$ *i.e* $\frac{4}{3}$, *or* 1·33.

Or, since the virtual image of the object has to be shifted farther away from it and on the same side of the lens, a convex lens will have to be used. Here $v=50$ cm. and $u=30$ cm. Calculate f of the lens.

85. An old person can see distinctly an object placed at a distance of 80 cm. from his eye. Required the focal length of a spectacle lens which will enable him to read a book held 36 cm. away from him. State the nature of the lens.

86. A long-sighted person whose nearest distance of distinct vision is 100 cm. finds that this distance is reduced

to 20 cm. by using a pair of spectacles. Find the dioptric power of the lens used.

87. Determine the focal length of a lens which a short-sighted person should use in order to see an object at a distance of 10 in; when he is able to see clearly at a distance of 6 in. without the glasses. What kind of lens is necessary?

88. A short-sighted person cannot see distinctly objects that are more than 10 ft. away. What kind of spectacles would he require to see clearly an object 100 ft. away? [B.A. 1929]

89. A pair of spectacles is made of two similar lenses each having two convex surfaces of 10 in. and 20 in. radius respectively. A person looking through them finds that the nearest point to which he can focus is one foot away from the glasses. What is the nearest point of the distinct vision without the spectacles? (${}_a\mu_g = 1{\cdot}5$) [B.A. 1928]

10. Simple microscopes and telescopes

Magnifying power m of a lens of focal length f is given by

(i) $m = 1 - \dfrac{D}{f}$, when the image is formed at the distance of distinct vision D.

and (ii) $m = -\dfrac{D}{f}$ when the image is at infinity.

Magnifying power of a telescope.

$$m = \frac{\text{angle subtended by the image at the eye}}{\text{angle subtended by the object at the eye}} = \frac{\beta}{\alpha}$$

$$= \frac{\text{distance of the real image from the objective}}{\text{distance of the real image from the eye piece}}$$

$$= \frac{F_o \text{ focal length of the objective}}{F_e \text{ focal length of the eyepiece}}$$

for normal adjustments.

90. A certain lens when held close to the eye magnified 5 times. What must be the focal length and the power of the lens, if the eye can see distinctly at a distance of 30 cm. ? [I.Sc. 1925]

Magnification $m = 1 - D/f$. $\therefore$ $30/f^{*} = 1 - 5$.

$\therefore$ $f = 30 / (1 - 5) = -7{\cdot}5$ cm., and the power of the lens is $(100/7{\cdot}5) = 13{\cdot}3$ dioptres.

91. Find the greatest magnification that can be produced by a converging lens of focal length 10 cm. Take the distance of clearest vision as 25 cm.

92. A watch maker's magnifying glass has a focal length of 5 cm. If the image is formed at a distance of 25 cm. away for distinct vision when the lens is held close to the eye, find its magnifying power.

93. To read a micrometer scale, a person whose distance of distinct vision is 30 cm. uses a magnifying lens of 8 cm. focal length. Find the magnifying power of the lens. Find also the distance of the scale from the lens. The lens is held close to the eye.

93a. The distance of distinct vision for an individual is 24 cm. and he uses a lens of focal length 12 cm. as a magnifying glass. Explain what will be the magnification, (*a*) if the image is formed at the distance of distinct vision and (*b*) if the rays coming through the lens are parallel. [I.Sc. 1932]

94. Two convex lenses, each of 20 in. focal length, are situated 10 cm. apart along a common axis. An object 2 in. high is placed 15 in. away from the first lens. Find the size and the position of the final image.

*Least distance of distinct vision is between 16 to 18 in. for a normal eye as per opticians though we usually take it between 10 to 12 in. for the calculations in Physics.

$u_1 = 15$ in. $f_1 = f_2 = -20$ in. The position of the first image v_1 will be given by $1/v_1 = 1/15 - 1/20$ $\therefore$ $v_1 = 60$ in. This image is at a distance of $(60+10=)$ 70 in. from the second lens and acts as an object to it; therefore $u_2 = 70$ in. Again from $1/v_2 = 1/70 - 1/20$, we get $v_2 = -28$ in. showing that the image is real and 28 in. beyond the second lens. The magnification m_1 due to the first lens $= I_1/O = 60/15$, and m_2 due to the second lens $I_2/I_1 = 28/70$.

$\therefore$ total magnification $m_1 \times m_2 = (4 \times 2/5 =)$ 1·6.

$\therefore$ $I_2 = (1{\cdot}6 \times 2 =)$ 3·2 inches.

95. A tree 50 ft. high is situated at a distance of 200 ft. from a convex lens of 20 in. focal length. Another lens of 4 in. focal length is placed at a distance of 2 ft. from the first lens along a common horizontal axis. Determine the sise of the final image of the tree as seen through both the lenses.

96. The objective lens of an astronomical telescope has a focal length of 30 in. and the eye-piece-lens a focal length of 2 in. Calculate the magnification that the telescope will produce when a final image of a distant object is seen (i) a long way off, (ii) at a distance of 12in. Find the distance between the two lenses in each case.

(i) The magnification $= F/f = 30/2 = 15$ and the distance between the two lenses is $(30+2=)$ 32 in.

(ii) $v_2 = 12$ in. $f_2 = -2$ in. To find u_2 the position of the first image from the eye-piece, we have $1/u_2 = 1/2 + 1/12$. $\therefore$ $u_2 = \frac{12}{7}$ in. As the object is at infinity, $v_2 = F$. $\therefore$ the magnification $= [30/(12 \div 7) =]$ 17·5 and the distance between the two lenses $= (30 + \frac{12}{7} =)$ 31·71 in.

97. A distant chimney is viewed through a telescope

consisting of two convex lenses. Trace the path through the telescope of rays from the top and bottom respectively of the distant object. If the lenses have focal lengths of 60 cm. and 4·0 cm. respectively, determine the magnification when the final image (*a*) is formed at infinity, (*b*) is formed at the least distance of distinct vision, *i.e.*, at 30 cm. [B.A. 1930.]

98. A telescope has two lenses of focal lengths equal to 2 ft. and 2 in. respectively. Find the magnification when it is used to view (*a*) a distant object, (*b*) an object at a distance of 40 ft. from the object glass.

Hint : in (*b*) find v_1; the magnification will be v_1 / f instead of F/f, considering the distance of the object great as compared to f.

99. An astronomical telescope, the lenses of which *have focal lengths 36 in. and 2 in., produces a real image* of the sun on a screen placed 1 ft. from the eye-lens. Calculate the size of the image observed. (Angular diameter of the sun is 0·009 radians.) [I.Sc. 1927]

Hint: Find u_2 the distance of the first image from the eye-lens then the magnification is F/u_2. Again α is given, hence find β in radians. $v_2 \times \beta$ gives the size of the image (β is the angle subtended by the image at the eye-piece.)

***100.** A 'reading telescope' has lenses of focal lengths 25cm and 5cm respectively. It is used to view an object 1 m. away from the objective. If the image is formed at a distance of 25 cm. from the eye which is *very close* to to the eye-piece, calculate the magnifying power of the telescope.

From the relation for lenses, we get $1/v_1 = 1/100 - 1/25$. Hence v_1 for the objective is $-\frac{100}{3}$. Therefore, the

magnification $m_1 = (\frac{100}{3} \times \frac{1}{100}) = \frac{1}{3}$. But the eye-piece acts as a magnifying lens giving the image at the clearest vision, hence, $m_2 = (1 - \frac{25}{5} =)6$ $\therefore$ the required magnifying power is given by $m_1 \times m_2 = \frac{1}{3} \times \frac{6}{1} = 2$. (Compare Ex. 96.)

***101.** A convex and a concave lens each 10 in. in focal length are held coaxially at a distance of 5 in. apart. Find the position of the image if the object is at a distance of 15 in. beyond (*a*) the convex lens (*b*) the concave lens.

(*a*) In absence of a concave lens, the convex lens would give a real image at a distance v from it.

From $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ we get $\frac{1}{v} = \frac{1}{15} - \frac{1}{10}$ $\therefore$ $v = -30$ in.

Now on interposing the concave lens, the image acts as an object to this lens $u' = (-30+5=)$ -25 in. (*Note the minus sign.*)

$\because$ $\frac{1}{v'} = \frac{1}{10} + \frac{1}{-25}$ $\therefore$ $v' = 16{\cdot}6$ in.

The final image thus is vertual and 16·6 in from the concave lens on the side of the object.

(*b*) For the concave lens, using the usual relation.

$\frac{1}{v'} = \frac{1}{10} + \frac{1}{15}$, we have $v' = 6$ in.

Hence for the convex lens, $u = (6+5 =)$ 11 in.

$\therefore$ from $\frac{1}{v} = \frac{1}{11} - \frac{1}{10}$, we get, $v = -110$ in.

The image is real and at 110 in. from the convex lens.

***102.** A convex lens of 20 cm. focal length and a concave lens of 4 cm. focal length are situated 16 cm. apart to form a Galileon telescope. Find the magnification when the object is (1) a long way off, (2) at a distance of 10 m. from the object glass. Find the position of the final image.

HEAT

1. Thermometry

(1) To change C readings to F readings of temperature, multiply a C reading by $\frac{9}{5}$ and add 32.

(2) To change from F to C temperatures, substract 32 from F reading and multiply by $\frac{5}{9}$.

(3) To change from C to A (absolute scale) temperatures, add 273 to a C reading.

(4) To change from F to A, first convert F to a corresponding C temperature then add 273.

The relation can be expressed as

(i) $\dfrac{F-32}{9}=\dfrac{C}{5}$ from (1) and (2)

(ii) $C+273=A$ from (3)

(iii) $\dfrac{(F-32)\,5}{9}+273=A$ from (4)

1. What Centigrade temperatures correspond to :—
(*a*) $50°F$ (*b*) $-40°F$ (*c*) $95°F$?

Applying the relation (i) we have

$$\frac{C}{5}=\frac{F-32}{9} \quad \therefore \frac{C}{5}=\frac{50-32}{9}$$

$\therefore C = 10°$ in (*a*)

$C=\dfrac{-40-32}{9}\times 5=-40°$ in (*b*)

$C=\dfrac{95-32}{9}\times 5=35°$ in (*c*)

2. Convert the following readings of a Centigrade thermometer into degrees of Fahrenheit scale:—

$5°C$, $-20°C$ and $-273°C$.

3. What are the readings on the Absolute scale of temperature corresponding to :—

$-40°F$, $-180°C$, $233°C$ and $104°F$?

4. What Centigrade temperatures are corresponding to $0°A$, $300°A$, $14°F$ and $212°F$?

5. An accurate Centigrade thermometer registers 25°, while at the same time a faulty Fahrenheit thermometer registers 76·55°. Determine the error* in the faulty thermometer.

Hint: First convert $25°C$ into degrees *F*., and thence compare the values in *F*. This will give the error.

6. A Fahrenheit thermometer indicates 90·5°, while a faulty Centigrade thermometer registers 32·55°. Ascertain the error in the latter.

7. The boiling point of water as marked by a thermometer is $99·6°C$ when the atmospheric pressure is 752 mm. What correction should be applied to the upper fixed point, if a fall of 10 mm. pressure lowers the boiling point of water by $0·37°C$? The *B.P.* of water at 760 mm. is $100°C$

Hint: Calculate the *B.P.* of water at the given pressure, and hence find the correction to be applied.

8. The freezing point of a certain liquid is given by the same number on the *F* and *C* scales. What is that temperature ?

---

* (i) The *correction* to be applied is the real temperature *minus* the observed reading.

(ii) All *errors* are subtracted from the actual readings while the *corrections* are added to the actual readings. In problem 5 above, error is $-0·45°F$ while the correction is $+0·45°F$.

Note the convention of algebraic signs as applied to '*errors*' and *corrections*.

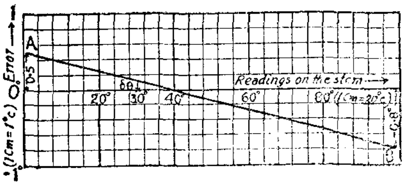
Let t be the required reading on both the scales, then we have $\frac{t-32}{9} = \frac{t}{5}$ *or* $5t = 9t + 160 \quad \therefore \; t = -40°$.

9. A thermometer with an arbitrary scale of equal parts reads 15 in melting ice and 240 in boiling water under standard pressure. Calculate the Centigrade temperatures indicated by the readings 10 and 85 on this thermometer. [B.A. 1929]

Hint : Obtain the values in C in a way similar to the conversion of F into C readings.

10. If when the temperature is $0°C$ a mercury thermometer reads $0{\cdot}5°C$ while at $100°C$ it reads $99{\cdot}2°C$, find the true temperature when this thermometer reads $30°C$ assuming the bore cylindrical and the divisions of uniform length. Verify the result by the graphical method.

$(99{\cdot}2 - 0{\cdot}5 =)$ $98{\cdot}7$ divisions between $F.P.$ and $B.P.$ on the stem represent $100°C$, hence $0{\cdot}987$ division is equivalent to $1°C$ on a correct thermometer. The temperature represented by 30th division corresponds to $(30 - 0{\cdot}5 =)$ $29{\cdot}5$ divisions on the scale above the $F.P.$ But $29{\cdot}5$ divisions represent a temperature difference of $(29{\cdot}5/0{\cdot}987 =)$ $29{\cdot}89$ degrees. Hence the true temperature is $29{\cdot}89°$ C.



Error Graph

or Mark the readings on the stem along the X-axis and errors at the observed fixed points along the Y-axis with their proper algebraic signs. Choose any convenient units. In the example, AB is the graph thus obtained. The error $\delta\theta$ at $30°C$ is $+0{\cdot}11°C$. $\therefore$ the

true temperature corresponding to a reading 30° is (30 − 0·11 =) 29·89°C. Similarly at 85°, the error is − 0·6° and the correct temperature is [85 − (− 0·6) =] 85·6°C.

11. Taking the bore of a thermometer and its graduations unifrom along the length of the stem, determine the correct temperature when it reads 20°C., if the fixed points indicated by it at a standard pressure are − 0·2° and 100·8°C respectively.

2. Expansion of solids

If l_1 and l_2 are the lengths of a solid at the temperatures t_1 and t_2 then $l_2 = l_1 \ [\ 1 + \alpha\ (\ t_2 - t_1\ )\ ]$ where α is the coefficient of linear expansion of the solid. (1)

$$or\ \alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}\ i.e. = \frac{\text{change in length}}{\text{original length} \times \text{change in temp.}} \qquad (2)$$

and the coefficient of superficial expansion of a solid $= 2\alpha$* (3)

while the coefficient of cubical expansion $= 3\alpha$ (4)

Following are the coefficients of linear expansion used in this section.

Brass 0·000019. Glass 0·000008. Copper 0·000017.
Silver 0·000019. Iron-steel 0·000011 Aluminium 0·000025.

12. A copper rod 100 cm. long at 0°C expands by 0·167 cm. when heated to 100°C. Determine the coefficient of linear expansion of the solid per degree Centigrade.

13. An iron steam-pipe 996 cm. long expands by 1 cm., when steam is turned on so that it is heated from 68°F to 212°F. Find the coefficient of linear expansion of iron per degree Fahrenheit.

* The relation is not strictly true. However, for all practical purposes it does not introduce any sensible error.

14. An iron bar 4 ft. long at $0°C.$, when heated in a furnace increases in length by 0·5 in. Determine the temperature of the furnace.

Let $t°C$ be the temperature of the furnace, then since $\alpha = \dfrac{l_t - l_0}{l_0 \times t}$, $t = \dfrac{0 \cdot 5}{12 \times 4 \times 0 \cdot 000011} = 947 \cdot 3°C.$

15. Two strips of equal width and thickness, one of iron and the other of brass are rivetted together at close intervals. Describe what happens to this compound bar on heating from $0°C$ to $500°C$. Find the length of each strip, if originally they measured one metre each. (a for iron 0·000012, α for brass 0·000019.)

16. A wheel is 4 m. in circumference. An iron tyre measures 3·995 m. around its inner face. How much must the temperature of the tyre be raised, so that it may just slip on the wheel ? ($\alpha_{Fe} = 0 \cdot 000011 /°C$)

Hint: The inside circumference of the tyre, when heated should just equal the circumference of the wheel.

17. A hollow steel cylinder of internal dimeter 25 in. is to be made to fit over another cylinder of diameter 25·1 in. To what temperature must the hollow cylinder be heated to make it just slip over the latter cylinder ? The air temperature is $30°C$.

18. The iron rails on a railway are each 10 ft. What space must be left between two consecutive rails, if the temperature may range from $20°C.$ to $40°C.$?

Hint: The minimum space, that should be kept, must equal the increase in 10 ft. of a rail, when the temperature rises by $20°C$.

19. When the temperature of air is $30°C$, a 50-yd. track is measured off with a 10-yd. steel tape correct at $20°C$. Determine the error in the length of the track.

Two scale divisions, whose distance apart is 1 yd. at $20°C$. will be at a distanee of 1 [1 + 0·000011 (30 − 20)] yd. apart at $30°C$. The length of the road is 50 of *these divisions.* Hence the real length of the road is

$$50\ (\ 1+0{\cdot}000011 \times 10\ )\ \text{yds.}$$

$$\therefore \text{ the error} = -50 \times 0{\cdot}000011 \times 10 = -0{\cdot}0055 \text{ yd.}$$

20. A mercury column measures 76 cm. when the temperature is $30°C$. Calculate its real length if the scale with which it is measured is graduated at $15°C$ and if it is given that the coefficient of linear expansion of the material of the scale is 0·00002. [I.Sc. 1929]

• **21.** A barometer reading is taken on a brass scale, the graduations on which are correct at $0°C$. What would be the true height of the barometer column which was read at $40°C$ by the scale as 30 in ?

• **22.** Two vertical walls were 20 ft. apart when built. These bulged out at their centres to the extent of 1 in. each. If they are to be made exactly vertical by the contraction of an iron bar, by how much must its temperature be raised above that of air ?

23. A pendulum is to be made of rods of steel and brass. The effective length of the pendulum is one metre and this length must remain constant when the temperature changes. How must the rods of steel and brass be chosen ? Draw a neat diagram of the pendulum

[B.A. 1920]

Hint : $\dfrac{\text{Total length of steel rods}}{\text{Total length of brass rods}} = \dfrac{\alpha \text{ of brass}}{\alpha \text{ of steel}}$.

• **24.** An aluminium piston 7 cm. in diameter slides in an iron cylinder at $30°C$. What must be the diameter of the cylinder at $30°C$, if the clearance between the piston and the cylinder is expected to be 0·1 mm. at $300°C$.

24. The area of a brass-plate is 100 sq. cm. at 20°*C*. Determine its area at 100°*C*.

Since area at 100°*C* is given by

$$A_{100} = A_{10}\,[\,1+2\alpha\,(t_{100}-t_{20})\,]$$
$$A_{100} = 100\,[\,1+2\times0{\cdot}000019\,(\,100-20\,)\,]$$
$$= 100{\cdot}304 \text{ sq. cm.}$$

26. A silver plate has an area of 200 sq. cm. What will it be at 60°*C*., if it is originally at 20°*C* ?

26*a*. A 3 ft. by 2 ft. steel door just fits in a frame of an all steel safe at 20°*C*. What will be the area of the door when the room temperature is 40°*C* ? Can the door fit the frame when the higher temperature of both is the same ?

27. A steel ball has a volume of 500 cc. at 10°*C*. What volume would it occupy on heating to 30°*C* ?

28. The capacity of a glass flask is 1000 cc. at 0°*C*. By how much will it increase when heated to 100°*C* ?

29. The density of a piece of a copper is 8·93 gm./cc. at 0°*C* and its coefficient of linear expansion is $16{\cdot}78\times10^{-6}$ per degree centigrade. What is its density at 100°*C* ?

Hint: Density varies inversely as the volume, therefore $d_t = d_o(\,1-3\alpha t)$. Determine the density at 100°*C* from this.

3. Expansion of liquids

Coefficient of expansion of a liquid is the increase of volume per unit volume per unit rise of temperature.

When a liquid expands on heating, the vessel which contains this liquid also expands. Therefore the expansion *of a liquid in a vessel* is called its apparent expansion. While the actual change of volume of the liquid is called the real expansion of the liquid. Hence we have got the following relation.

The apparent coefficient fo expansion α of a liquid is equal to the real coefficient of expansion r minus the coefficient of cubical expansion β of the solid envelope, *i.e.* .

$$r = \alpha + \beta \qquad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \qquad (1)$$

The above relation is only approximate, in fact

$$r = \frac{V_2 - V_1}{V_1 \, (t_2 - t_1)} + \frac{V_2}{V_1}\beta = \alpha + \frac{V_2}{V_1}\beta \qquad \cdot \qquad \cdot \qquad (2)$$

The relation (2) can be expressed in terms of weights so that $r = \dfrac{w_1 - w_2}{w_2 \, (t_2 - t_1)} + \dfrac{w_1}{w_2}\beta = \alpha + \dfrac{w_1}{w_2}\beta \quad \cdot \quad \cdot \quad (3)$

30. A glass sp.gr. bottle weighed 40 gm. when empty and 176·1 gm. when filled with mercury at $0°C$. Find the mass of mercury which will be expelled when the bottle is heated to $100°C$. Coefficient of apparent expansion of mercury is 0·00015 per degree centigrade.

Weight of mercury at $0°C$ in the bottle is (176·1 − 40 =) 136·1 gm. Let w_2 be the weight in it at $100°C$. We want to find the weight of mercury expelled for which w_2 will have to be determined. Hence using the relation

$$\alpha = \frac{w_1 - w_2}{w_2 \, (t_2 - t_1)} \text{ we have } 0{\cdot}00015 = \frac{136{\cdot}1 - w_2}{w_2 \times 100}.$$

$$\therefore \quad 0{\cdot}015\, w_2 = 136{\cdot}1 - w_2 \,; \; or \; 1{\cdot}015\, w_2 = 136{\cdot}1.$$

$$\text{Hence} \quad w_2 = 134{\cdot}1 \text{ gm.}$$

$\therefore$ the mercury expelled = (136·1 − 134·1 =) 2 gm.

31. A sp.gr. bottle when empty weighs 20·32 gm. When filled with a certain liquid and heated to $40°C$, it weighs 81·73 gm. Determine the mass of the liquid which will be expelled on heating it to 60°C. The coefficient of apparent expansion of the liquid is $46{\cdot}84 \times 10^{-5}$ per degree C.

32. The bulb of a mercury thermometer is one cubic

centimetre and the section of the bore of its tube is 0·001 sq.cm. Find the position of the mercury at $60^\circ C$. if it just fills the bulb at $0^\circ C$.

Hint: Find the apparent volume of mercury at 60° from $v_{60} = v_0 [1 + (r - \beta) t]$. The increase over the original volume gives the length over which it spreads in the stem.

33. To measure temperatures between $-30^\circ C$ and $200^\circ C$ a mercury thermometer is to be constructed. What should be the capacity of the bulb at $0^\circ C$, if the bore of the stem has the cross-section of 0·001 sq. cm. and the divisions on the stem etched at a distance of 2 mm. each ?

34. Find the true temperature of a liquid which when boiling under normal pressure shows a temperature of $287 \cdot 9^\circ C$ by a thermometer the stem of which above zero has an average temperature $20^\circ C$ and the rest of it is in the boiling liquid.

Let x be the true boiling point of the liquid. Since $(287 \cdot 9 - 0 =)$ 287·9 of the stem are exposed to a temperature of $20^\circ C$, mercury in the stem up to 287·9° would have got expanded by $[287 \cdot 9 \times 0 \cdot 00015 (x - 20)]$ if the whole stem were at $x^\circ C$.

$\therefore \quad x = 287 \cdot 9 [1 + 0 \cdot 00015 (x - 20)] \quad \therefore \quad x = 300^\circ C.$

35 What will be the reading on an accurate mercury thermometer, if the bulb and the stem, upto $0^\circ C$ are at $200^\circ C$ and the rest of the stem at an average temperature of $30^\circ C$.

36. Pure water is boiling under normal pressure in a flask in which a thermometer with its bulb and stem upto $0^\circ C$ mark is inserted. If the rest of the stem is at $30^\circ C$, what will be the boiling point of the water as registered by the thermometer ? [I.Sc. 1927]

37. For determining the absolute expansion of a liquid by Dulong and Petit's apparatus the hot column was found to be 76·9 cm, high at $100°C$, while the cold one 74·5 cm. high at $23°C$. Calculate the coefficient of expansion of the liquid.

Hint: The difference in the heights is (76·9 – 74·5 =) 2.4 cm., the length of the hot column before expanding is 74·5 cm. Hence calculate r the increase in volume per unit volume per degree C. Cross-sections of the tubes are uniform.

38. One limb of a U-tube containing a liquid is sur rounded by water at $4°C$ and the other by steam at $100°C$. If the cold column measures 60 cm , find the height of the other. Absolute expansion of the liquid = 0·000061 per $1°C$.

39. In a Regnault's apparatus for absolute expansion of liquids the difference in levels of the liquid in the arms of the T-piece was found to be 0·9 cm., while the height of the hot column was 50·0 cm. Calculate the coefficient of expansion of the liquid, the temperature difference between the two columns being $100°C$.

Hint: The effective height of the cold column is (50·0 – 0·9 =) 49·1 cm., and 0·9 cm. is equivalent to the increase inl ength.

40. A glass vessel of 180 cc. capacity is filled with mercury in such a way that when both are heated together to the same temperature the volume of the empty part remains unaltered. Find the volume filled up by the mercury. Coefficient of cubical expansion of glass is 0·000024.

Hint: Increase in the capacity of the vessel must just equal the true increase in the volume of mercury in it when both are heated.

41. A weight thermometer of glass contains 10 gm. of mercury at $0^\circ C$ and 9·843gm. at $100^\circ C$. Calculate the coefficient of absolute expansion of mercury, that of glass being 0·000022. [I.Sc. 1914]

42. Calculate the coefficient of expansion of the glass from the following data : —

Wt. of an empty thermometer	=	50 gm.
,, ,, + mercury filling it at $0^\circ C$	=	710 gm.
,, ,, ,, ,, ,, $100^\circ C$	=	700 gm.
Coefficient of absolute expansion of mercury	=	0·000018

[I.Sc. 1915]

43. A solid weighs 42·780 gm. in air ; when immersed in a liquid at $20^\circ C$, it weighs 37·680 gm. ; when immersed in the same liquid at $70^\circ C$ it weighs 37·765 gm. If the coefficient of linear expansion of the solid is 0·00002, calculate the coefficient of real expansion of the liquid. [I.Sc. 1928]

The mass of the liquid displaced at 20° is $(42{\cdot}780 - 37{\cdot}680 =)$ 5·1 gm. which is equal to $V_{20}\rho_{20}$ where V_{20} is the volume of solid at 20° and ρ_{20} the density of the liquid at that temperature.

Similarly the mass of the liquid displaced at 70° is $(42{\cdot}780 - 37{\cdot}765 =)$ 5·015 gm.. which is equal to $V_{70}\rho_{70}$.

$$\text{But } V_{70}\rho_{70} = V_{20}[1+3\times2\times10^{-5}(70-20)]\frac{\rho_{20}}{1+r\,(70-20)}$$

$$\therefore \quad 5{\cdot}015 = \frac{5{\cdot}1\,(1+6\times10^{-5}\times50)}{1+50\,r}.$$

$$\text{Hence } \frac{1+50\,r}{1+3\times10^{-3}} = \frac{5{\cdot}100}{5{\cdot}015} \quad \therefore \; r = 0{\cdot}0004$$

or using the relation (3) we get

$$r = \frac{5{\cdot}1 - 5{\cdot}015}{5{\cdot}015\times50} + \frac{5{\cdot}1}{5{\cdot}015}\times 6\times10^{-5} \quad \therefore \; r = 0{\cdot}0004$$

44. Calculate the coefficient of linear expansion of a solid from the following data :—

Weight of the solid in air		$= 94{\cdot}200$ gm.
,,	,, in a liquid at $20^\circ C$	$= 44{\cdot}200$,,
,,	,, in the liquid at $80^\circ C$	$= 46{\cdot}822$,,

Coefficient of real expansion of the liquid is 95×10^{-5}.

45. If a Fortin's barometer reads 75 cm. at $30^\circ C$, determine the correct reading of the barometer. The linear coefficient of scale-metal is 0·00002. [B.A. 1921]

The corrected height isegiven by the relation $H_0 = H\ [\,1 + (\beta - r)\ (t_2 - t_1)\,]$. Here $t_2 - t_1 = 30$, and $\beta - r = -0{\cdot}00016$. $\therefore\ H_0 = 75\ (1 - 0{\cdot}00016 \times 30) = 74{\cdot}64$ cm.

4. Expansion of gases

In case of gases, expansion of the containing vessel is small compared to that of a gas and hence generally it is neglected.

When a gas is heated (1) its pressure increases if its volume is kept constant, and (2) the volume expands if the pressure is maintained constant.

Therefore the following relations hold for the gases :—

(*a*) *Charles' law*—Expansion at constant volume.

$$P_t = P_0\,(1 + \alpha t)^\dagger \cdots \text{Constant volume law} \quad (1)$$

$$\frac{P_2}{P_1} = \frac{1+\alpha t_2}{1+\alpha t_1} = \frac{T_2}{T_1} \text{ and } \alpha = \frac{P_2 - P_1}{t_2 \times P_2 - P_2 \times t_1} \qquad (2)$$

(*b*) *Gay-Lussac's law*-Expansion and constant pressure.

$$V_t = V_0\,(1 + \alpha t)^\dagger \cdots \text{Constant pressure law} \quad (3)$$

$$\frac{V_2}{V_1} = \frac{1+\alpha t_2}{1+\alpha t_1} = \frac{T_2}{T_1} \text{ and } \alpha = \frac{V_2 - V_1}{t_2 \times V_1 - V_2 \times t_1} \qquad (4)$$

† The coefficient of expansion of a gas is so large that it is very important that V should be taken as the volume at $0^\circ C$ and P_0 as the pressure at $0^\circ C$.

(c) $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$, Gas equation for a perfect gas (5)

When T_1 and T_2 are the temperature on the absolute scale and t_1 and t_2 are the corresponding temperatures on the Centigrade scale, V, P, and α have usual meanings.

46. 300 cc. of air at 27°C exert a pressure of 800 mm. of mercury. What pressure can it exert, when heated to 177°C, if the volume is maintained constant ?

Here the absolute temperature $T_2 =$ (273+177=) 450°
and ,, ,, $T_1 =$ (273+27 =) 300°
and the original pressure $P_1 =$ 800 mm.

As the volume is to remain constant to find the new pressure P_2, we get, from Charles' law

$$P_2 = \frac{P_1T_2}{T_1} = \frac{800\times 50}{300} = 1200 \text{ mm. of the mercury.}$$

47. A cycle tube was inflated with air at 27°C and the pressure was 24 lb./sq. in. Being kept in the sun, it burst. What must have been its temperature; if it was made to withstand a pressure of 26·4 lb./sq. in. ?

48. In an experiment with an air thermometer (constant volume pattern) the pressure noted at 20°C was 81·9 cm. of mercury. On heating the bulb to a certain temperature, the pressure shown by it was 93·6 cm. of mercury. Neglecting the expansion of glass, determine the higher temperature.

49. A certain amount of gas occupies a volume of 1000 cc. at 0°C and 2 litres on heating it to 273°C. Determine the coefficient of expansion of the gas at constant pressure.

If α be the coefficient of expansion, since the volume is considered at 0°C we have $V_{273} = V_0 (1+\alpha t)$

$$\therefore \quad 2000 = 1000 (1+\alpha\times 273)$$

$$\therefore \quad 1+273\,\alpha = 2, \ or \ \alpha = \frac{2-1}{273} = 0{\cdot}00366.$$

50. A hollow copper sphere, 40 cm. in diameter, is filled with air at 27°*C*, under a pressure of 2 atmospheres and hermitically sealed. On heating it to 181°*C*, the pressure in the sphere rises to 3 atmospheres. Calculate the coefficient of pressure of the air at constant volume. Neglect the expansion of the sphere.

Hint: Use the relation 2 and not 4. See the foot-note on page 125.

51. In a constant volume gas thermometer, the pressure at °*C* is 546 mm. and that at 100°*C*, is 74·4 cm. If the volume coefficient of expansion of glass is 0·00003, find the coefficient of increase of pressure of gas.

[I.Sc. 1926]

Since $\alpha - \beta = \dfrac{P_2 - P_1}{P_1 \times t_2 - P_2 \times t_1}$, if β is not neglected.

On introducing the known values for the symbols P, t and β we get, $\alpha - 0{\cdot}00003 = \dfrac{744 - 546}{546 \times 100 - 744 \times 0}$

$$\therefore \quad \alpha = \tfrac{198}{54600} + 0{\cdot}00003 = 0{\cdot}003657.$$

52. The volume of a flask is 1000 cc. at N.T.P. The flask is heated to 100°*C*. Find the volume of the air at N.T.P. still remaining in the flask, if the coefficient of expansion of air is 0·0037. [I.Sc. 1919]

Hint : The required volume of air+the increase in that volume at 100°*C*, equal the total capacity of the flask at 100°*C*.

53. Find the temperature at which a litre of a gas originally at 27°*C*, would occupy 1500 cc., the pressure remaining constant.

Applying Gay-Lussac's Law $V \propto T$ we have, on substituting the values of $V_1 = 1000$ cc., $V_2 = 1500$ cc. and

$$T_1 = (\ 273+27= \ ) \quad 300^\circ A, T_2 = \frac{1500 \times 300}{1000} = 450^\circ A,$$

Hence the required temperature

$$t_2 = (\ 450 - 273 = \ ) \ 177^\circ C.$$

54. 10 litres of air at $27^\circ C$ are enclosed in the cylinder of an air engine the cross-section of which is 100 sq. cm. The piston is allowed to move through a distance of 10 cm. to compress the air. If the pressure remains constant, find the change in temperature.

55. A litre of gas at $27^\circ C$ was heated to $327^\circ C$ at a constant pressure. Find the change in volume.

56. To find the volume coefficient of air at constant pressure the following observations were noted :—

The weight of the globe full of dry air		45·375 gm.
,, ,, ,,	water	385·375 gm.

After removing water from it, it was heated to $98^\circ C$ in a water bath, and then was just inverted in a tub of water so that the inside level of water was the same as that outside one when cooled to $15^\circ C$. The water drawn into the flask at $15^\circ C$ measured 75 cc. Neglecting the pressure of water vopour and the expansion of the globe, calculate the *coefficient of volume* expansion.

The capacity of the globe = (385·375 – 45·375 =) 340 cc. Hence, as the expansion of the globe is neglected, 340 cc. of air at $98^\circ C$ occupy (340 – 75 =) 265 cc, at $15^\circ C$. Therefore, uisng Gay-Lussac's Law we get

$$\alpha = \frac{343 - 265}{98 \times 265 - 340 \times 15} = 0{\cdot}00359$$

57. A narrow tube, of a uniform bore closed at one

end, contains an air column at the bottom enclosed by a small thread of mercury at the open end. The tube originally at 25°C is heated in an oil-bath till the length of the air column changes from 4 cm. to 6 cm. Determine the temperature of the bath, and hence the volume coefficient of air.

58. A quantity of a gas measured 1092 cc. at N.T.P. Find its volume at 27°C and a pressure of 750 mm. of mercury.

Here, P, V, T, all change. We have P_1 V_1 and T_1 respectively equal to 760 mm., 1092cc. and 273°A; while P_2 = 750 mm. and T_2 = (273+27 =) 300°A. Hence using the gas equation, we get

$$V_2 = \frac{760 \times 1092}{273} \times \frac{300}{750} = 1216 \text{ cc.}$$

59. What mass of air is contained in a room $10 \times 5 \times 5$ metres in size, the barometric pressure being 740 cm. and the temperature 27°C? Density of air at N. T. P 1·293 gm. per litre. Neglect the presence of water vapour.

60. A litre of oxygen, at 27°C and under a pressure of 700 mm. of mercury, is subjected to a temperature of 57°C and a pressure of 1000 mm. of mercury. Find the alteration in the volume.

61. Find the pressure exerted by a litre of air at 27°C, if the same mass of the air exerts a pressure of 75 cm. of mercury at 280°A and occupies 700 cc.

62. A cylinder contains 6 cu ft. of a gas at 15 lb. wt. per sq. in. and 20°C. The amount of the gas in the cylinder is kept constant and the pressure is changed to 150 lb. wt. per sq. in. What should be the temperature of the gas if the volume is found to be 2·5 cu. ft.? [I. Sc. 1932]

*63. 800 cc. of hydrogen gas were collected in water voltameter at 27°C when a current of electricity was passed through it, the atmospheric pressure being 76 cm. of mercury. Next time when the barometer reads 72 cm., 909 cc. of the same gas at 30°C were collected in the voltameter on passing through it another current for the same time as before. Compare the quantities of the gas evolved neglecting the vapour pressure of water.

Hint: Reduce the volume to S. T. P. This will give the comparison of the quantities evolved.

64. Find the value of R in *c.g.s.* units in the case of hydrogen at a pressure of 10^6 dynes per sq. cm., its density being 0·0896 gm. /lite. [B. Sc. 1919.]

The density per cubic centimetre is 0·0000896 gm.

$$\therefore \text{ from } R = \frac{PV}{T} = \frac{P}{\rho T}, \text{ we get, } R = \frac{10^6}{8{\cdot}96 \times 10^{-5} \times 273}$$

$$\therefore \quad R = 4{\cdot}085 \times 10^7 \text{ ergs per gram per degree.}$$

Compare the answers in Examples 64 and 65. Account for the difference. Also find the value of R in 65 in litre-atmospheres.

65. Find the value of gas constant in gram-molecule calories given that 2 gm. of hydrogen at N. T. P. occupy a volume of 22400 cc. and J = 4·2 Joules. [I. Sc. 1936]

5. Calorimetry

(1) **A Calorie** is the amount of heat required to raise the temperature of 1 gm. of water through one degree at 15°C.

(2) **Specific heat** of a substance is the heat required by 1 gm of the material to raise its temperature through 1°C, *or* it may be taken as the thermal capacity per unit mass.

(3) **Thermal capacity** of a substance is the amount of heat which it would require to raise its temperature by 1°C.

(4) **Water-equivalent** of a body is the weight of water in grams, which would require the same amount of heat to raise its temperature through $1^\circ C$ as the body would require to have the same rise in temperature. Hence it equals the product "*mass of the body* $\times$ *sp. heat.*" This also is numerically equal to the thermal capacity of the body.

66. How many calories of heat are required to heat a laundry iron (sp. ht. 0·12) weighing 4 kg. from $30^\circ C$ to $130^\circ C$?

1 gm. of iron requires 0·12 cal. for $1^\circ C$ rise.

$\therefore$ 4 kg. ,, will require $(0{\cdot}12 \times 4000)$ cal. ,, ,,

$\therefore$ 4 kg. ,, ,, $[480\,(130-30)]$ cal. for $100^\circ C$,,

Heat required $= 48000$ calories.

67. 50 gm. of water at $90^\circ C$ are mixed with 30 gm. of water at $30^\circ C$. What is the temperature of the mixture, if no heat is lost in heating the vessel?

Let $t^\circ C$ be the temperature of the mixture then the heat lost by the hot water $= [(90-t)50]$ cal.
the heat absorbed by the cold water $= [(t-30)\,30]$,,
Assuming the heat gained by the cold water is all received from the hot water,

$$(90-t)50 = (t-30)30 \quad \therefore \; t = 67{\cdot}5^\circ C.$$

68. A copper calorimeter weighing 47·22 gm. contained 45·5 gm. of water at $28{\cdot}2^\circ C$. A quantity of water at $58{\cdot}2^\circ C$ was added to it and the resulting temperature was found to be $41{\cdot}3^\circ C$. If the calorimeter with its contents weighed 131·5 gm, determine the water-equivalent of the calorimeter, and hence the specific heat of its material.

69. A copper* calorimeter with a stirrer weighs 41·27 gm. A piece of a metal weighing 26·88 gm. initially

*Take the sp. ht. of copper as 0·1 unless stated to the contrary

at 90·5°C is introduced in the water in the calorimeter, and the temperature of the water, thereby, is found to rise from 29°C, to 30·5°C. Determine the specific heat of the metal-piece, if the water in the calorimeter weighs 39·05 gm.

Let S be the sp. ht. of the metal, then

$$\left.\begin{array}{l}\text{Quantity of heat}\\ \text{given out by it}\end{array}\right\} = \begin{array}{l}\text{mass} \times \text{sp. ht.} \times \text{fall in temp.}\\ [\,26{\cdot}88 \times S \times (\,90{\cdot}5 - 30{\cdot}5)\,]\text{cal.}\end{array}$$

$$\left.\begin{array}{l}\text{Heat gained by the water}\\ \text{in the calorimeter}\end{array}\right\} = \begin{array}{l}\text{mass of water} \times \text{rise in tem.}\\ [\,39{\cdot}05 \times (30{\cdot}5 - 29)\,]\text{ cal.}\end{array}$$

$$\left.\begin{array}{l}\text{Heat gained by the calor.}\\ \text{and stirrer}\end{array}\right\} = \begin{array}{l}\text{w.e. of calor} \times \text{rise in temp.}\\ [41{\cdot}27 \times 0{\cdot}1\,(\,30{\cdot}5 - 29\,)\,]\text{ cal.}\end{array}$$

Hence $26{\cdot}88 \times S \times 60 = 39.05 \times 1{\cdot}5 + 41{\cdot}27 \times 0{\cdot}1 \times 1{\cdot}5$

$$\therefore \quad S = \frac{(\,39{\cdot}05 + 4{\cdot}127\,)\ 1{\cdot}5}{60 \times 26{\cdot}88} = 0{\cdot}0415.$$

70. 200 gm. of a liquid (sp.ht. 0·6) are poured into a copper calorimeter which is heated to 100°C. If the initial temperature of the liquid is 30°C, determine the final temperature. The weight of the calorimeter is 200gm.

71. A piece of platinum(sp.ht. 0·032) weighing 50 gm. is taken from a furnace and plunged instantly into 200 gm of water at 30°C. If the temperature of the water rises to 42°C, what is the temperature of the furnace? Neglect the heat used up by the containing vessel.

72. A copper calorimeter weighing 108 gm. contained 87 gm. of water at 22·9°C. A piece of ice was introduced into the calorimeter and the temperature of the mixture after the ice hand just melted was found to be 17·9°C. Find the latent heat of fusion of ice, if the calorimeter with its contents weighed 200 gm.

73. A piece of iron (sp. ht. 0·12) weighing 200 gm. was heated to 100°C and dropped into a cavity in a block of ice. The quantity of ice melted was found to be 30 gm.

Determine the latent heat of ice.

Let L be the latent heat of fusion of ice, then the heat taken up by 30 gm. of ice in melting $=(30 \times L)$ cal. Again the heat supplied by the iron piece is given by

$$m \times S\,(t_1 - t_2) = 200 \times 0{\cdot}12\ (100 - 0) = 2400 \text{ cal.}$$

$$\therefore\quad 30 \times L = 2400 \text{ and and } L = 80 \text{ cal.}$$

74. A lump of copper weighing 400 gm. is heated in an oil-bath and then put instantly in a cavity in a block of ice. If 120 gm. of ice are found to be melted, find the temperature of the bath. (Latent heat of fusion of ice 80 cal/gm.)

75. 15·7 gm. of solid paraffin wax (sp.ht. 0·7) at 20°C were kept in a copper calorimeter weighing 90 gm. On adding to this 100 gm. of water at 66·5°C.,the temperature of the calorimeter was found to be 54°C after all the wax had just melted. Calculate the latent heat of fusion of paraffin. The melting-point of paraffin is 54°C.

76. How many calories of heat are set free when a tank, containing one ton of water, cools from 20°C to the freezing point and the water gets converted into ice? (Latent heat of ice is 80 calories per gm. 1 lb =453·6gm.)

77. 300 gm. of water at 40°C, 6 gm. of steam at 100°C and 20 gm. of ice at 0°C are mixed together. Find the resulting temperature when all the ice is just melted.

Here we reduce the contents of the calorimeter to the transition temperature *viz.* 0°C and then see if there if any excess of heat or lack of it over and apply this heat to the entire contents to determine the final temperature.

Heat content in 300gm. of water $= (300 \times 40 =)$ 12000 cals.
Heat content in 6 gm. of steam $= (6 \times 536 + 6 \times 100 =)$ 3816 cals.

Hence total heat available at $0°C$ = ($12000+3816$ =) 15816 cals.

Heat absorbed by the ice in melting = (20×80 =) 1600 cals.

Hence($15816 - 1600$ =)14216 cals.are left over to raise the temperature of 326 gm. of the mixture to $t°C$ above $0°C$.

$\therefore \quad 326t = 14216, \quad \therefore t = 43{\cdot}6°C$, the final temperature.

78. 100 gm. of ice at $-12°C$ are introduced into a mass of water at $0°C$ and thereby 7·2 gm. of water are frozen, the temperature of ice rising to $0°C$. Find the latent heat of fusion of ice, its sp. ht. being 0·5.

Amount of heat given out by 7·2 gm. of water in freezing is $7{\cdot}2\ L$ cal. The quantity of heat used by the ice in getting heated to $0°C$ = ($100 \times 12 \times 0{\cdot}5$ =) 600 cal.

$\therefore \ 7{\cdot}2\ L = 600$ or $L = 83{\cdot}33$ cal.

78a. 300 gm. of ice are added to a litre of water contained in a vessel of water-value 120 gm. Assuming the initial temperature of the water to be $20°C$, find the amount of ice unmelted. How much steam at $100°C$ must be then blown into the mixture to raise the temperature to $20°C$? (Latent heat of ice 80 cal / gm.) and that of steam 536 cal./ gm.)

79. Lead melts at $327°C$. Its latent heat of fusion is 5 cal. per gm. Taking its specific heat as 0·03, find the quantity of heat required to raise the temperature of 10gm. of lead from $27°C$ to its melting point $327°C$ and to melt it.

80. A porous jar of water-value 200 gm. contains 4 litres of water at $30°C$. How mueh water must evaporate so that the remaining wonld be at $20°C$? The latent heat of vaporisation of water at $30°C$ is 578 cal./gm.

81. The specific gravity of ice is 0·917. 10 gm. of metal at $100°C$ are immersed in a mixture of ice and water

and the volume of the mixture is found to be reduced by 125 c. mm. without change of temperature. Find the specific heat of metal. [Pat. Univ.]

82*. 50 gm. of water at $16°C$, placed in the tube of a Bunsen's ice calorimeter, move the mercury in the capillary through 20 cm. 5 gm. of a metal at $100°C$ move it similarly through 5 cm. Calculate the sp. ht. of the metal. [B. A. 1917]

20 cm. change in length is brought about by (50×16)cal.

$\therefore$ 5 cm. change in length will require $(5 \times \frac{800}{20} =)$ 200 cal

But this amount of heat is given out by 5 gm. of the metal in cooling through $100°C$. If s be its sp. ht.

we have, $5 \times s \times 100 = 200 \quad \therefore s = 0{\cdot}4$

6. Conductivity of solids.

The quantity of heat Q flowing in time t sec. across A sq. cm. of a slab of a substance of thickness l cm. and having the opposite faces at temperatures $\theta°_1$ C and $\theta°_2$ C is given by

$$Q = \frac{k\ (\theta_1 - \theta_2)\ A\ t}{l}$$

where k is the thermal conductivity of the substance.

Thermal conductivites generally are measured in cal. per sec. per cm. per degree C unless stated to the contrary.

83. A cylindrical metal rod 14 cm. long and of 3·88 cm. diameter has one of its ends surrounded by steam. Water at $25°C$ is allowed to steadily flow round the other end at 230 cc. per minute. Two thermometers inserted in the holes in the rod 10cm. apart read $81{\cdot}7°C$ and $57{\cdot}0°C$ respectively. Calculate the conductivity of the metal, if the temperature of the water flowing rises to $32°C$.

The cross-section of the rod $= (\pi \times 1{\cdot}94^2 =)$ 11·82sq.cm. while the temperature difference between the ends at a

distance of 10 cm. is $(81{\cdot}7-57{\cdot}0=)\ 24{\cdot}7^\circ C$, The heat flowing across the rod per second is $k\times 24.7\times 11{\cdot}82\times \frac{1}{10}$ where k is the conductivity of the metal.

The water collected per second is $(\frac{230}{60}=)$ 3·83 cc. *or* 3·83 gm. The quantity of heat taken up by this water is $[3{\cdot}83(32-25)=]$ 26·79 calories.

$\therefore\ k\times 24{\cdot}7\times 11{\cdot}82\times \frac{1}{10}=26{\cdot}79.$

$$\therefore\ k=\frac{26{\cdot}79\times 10}{24{\cdot}7\times 11{\cdot}82}=0{\cdot}9187 \text{ cal./ sec. / cm./}1^\circ C.$$

84. Find the quantity of heat lost per sq.m. per min. by a man clothed in fabric 0·2 cm. thick, if the room temperature be $27^\circ C$ and that of the body $37^\circ C$. (The conductivity of the fabric is $1{\cdot}22\times 10^{-4}$ cal./ sec./cm./ degree C.)

85. An iron boiler 1·25 cm. thick contains water at atmospheric pressure. The heated surface is 2·5 sq. metres in area and the temperature of the underside is $120^\circ C$. If the thermal conductivity of iron be 0·2 and the latent heat of evaporation of water 536, find the mass of water evaporated per hour. [Pat. Univ.]

86. A circular metal plate of radius 5·65 cm. and 0·5 cm. thickness has the whole of one face touching the melting ice and the other face in contact with steam. The coefficient of conductivity of the metal is 0·14. Find the quantity of ice melted per minute. [B. A. 1915]

Hint : First calculate the amount of heat emerging per minute at the face touching the ice. This heat is utilized in melting the ice at $0^\circ C$, Latent heat of ice is 80 cal. per gm.

7. Vapour Pressure and Humidity.

(1) Pressure of vapour which saturates a given space is the same for the same temperature whether there are other gases present or not.

(2) Each gas or vapour enclosed in a given volume fills the whole space and exerts the pressure which it would exert if it were alone occupying the whole space. The pressure exerted by each gas is called its partial pressure.

(3) Total pressure exerted by gases and vapours confined in the same space equals the sum of the partial pressure of all these gases and vapours,

$$\text{Relative humidity} = \frac{\text{mass of wat. vap in a given vol. of air}}{\text{mass of wat. vap. reqd. to saturate the same vol. of air at that temp.}}$$

$$\textit{or}\quad ,, \quad ,, \quad = \frac{\text{vap. pr. of water vap. in the air}}{\text{saturation vap. pr. at the same temp.}}$$

$$= \frac{\text{saturation vap. pr. at the dew-point}}{\text{saturation vap. pr. at room temp.}}$$

87. 50 cc. of oxygen saturated with water vapour are collected at a pressure of 740 mm. and $27°C$. Determine the volume of dry oxygen at N.T.P. Vapour pressure of water at $27°C$ = 26·5 mm.

Pressure P_2 exerted by 50 cc. of the dry oxygen at $27°C$ = (740 - 26·5 =) 713·5 mm. The volume V_o at N.T.P. of the oxygen is given by the relation,

$$\frac{P_oV_o}{T_o} = \frac{P_2V_2}{T_2} \therefore V_o = \frac{713{\cdot}5 \times 50 \times 273}{(273+27)\ 760} = 42{\cdot}72 \text{ cc.}$$

88. A gas is collected over water in a 100 cc. tube and measured 72·8 cc., the temperature and pressure of the surrounding atmosphere being $25°C$ add 74·39 cm. respectively. Calculate the volume of the gas at N. T. P. (V. P. of water at $25°C$ = 23·45 mm.) [I.Sc. 1927]

Note: Assume the cross-section of the tube to be uniform and the level of the water inside and outside to be the same.

89. A quantity of hydrogen gas is collected in an inverted vessel over water, and the observed volume is 190 cc. The barometer stands at 756·5 mm. and the level of the water in the inverted vessel is 27·2 cm. above that of the water in the trough, the temperature of water being 27°*C*. Calculate the mass of hydrogen gas V.P. of water at 27°C is 26·5 mm. and the density of hydrogen at N T.P. is 0·0896 gm./litre. [B.A. 1914]

90. Calculate the mass of the vapour enclosed along with the gas in the Example 87. Density of saturated vapour at N.T.P. is 0·8081 gm./litre.

Since the volume occupied by vapour at 27°C is 50 cc. and exerts a pressure of 26·5 mm., its value at N.T.P.

$$V_\circ = \frac{50 \times 26{\cdot}5}{(273+27)} \times \frac{273}{760} = 1{\cdot}586 \text{ cc. } \textit{or } 0{\cdot}001586 \text{ litres.}$$

$\therefore$ mass of the vapour = (0·001586 × 0·8081 =) 1·282 mg

91. Calculate the humidity of the atmosphere from the following data :—

The temperature of the atmosphere . . . . = 27°*C*.
,, ,, at which dew appeared . . = 9·5°*C*.
,, ,, ,, ,, disappeared = 10·5°*C*.

(·V. P. of water at 10°*C* = 9·14 mm. of mercury and at 27°*C* = 26·5 mm.)

Dew-point = ½ (9·5 + 10·5) = 10°*C* and room temp. = 27°*C*.

$$\therefore \text{ Humidity} = \frac{\text{S. V. P. at } 10^\circ}{\text{S. V. P. at } 27^\circ} = \frac{9{\cdot}14}{26{\cdot}5} = 0{\cdot}3448 \textit{ or } 34{\cdot}48\ \%.$$

92. Calculate the hygrometric state of air from the following observations taken with a chemical hygrometer.

Weight of U-tubes at first = 70·50 gm.
,, ,, ,, with water vapour from the ordinary air = 70·63 gm.
,, ,, ,, with water vapour

from the ordinary air plus the water
vapour from saturated air = 70·90 gm.

Hint : Find the amount of the vapour actually present in a given volume of air and that required to saturate the same volume of the air.

93. What difference in volume would be caused in a litre of dry air (free to expand or contract) at 700 mm. pressure and 20°*C*., if sufficient water to saturate the space were introduced without altering the pressure and the temperature ? *S.V.P.* of water at 20°*C* = 17·5 mm.

94. A cubic metre of air the relative humidity 80% is dried at 30°*C* under the atmospheric pressure of 75·5 cm. Find the change in the volume B. P. of water at 32 mm. pressure is 30°*C*.

***95**. The dimensions of a hall are 30 by 15 by 5 metres. Calculate the increase in the mass of water vapour present in it, if the humidity increases from 0·4 to 0·6 while the temperature remains unaltered. The density of the saturated water vapour at the temperature of the hall is 0·0008 gm. per cc. [I.Sc. 1930]

Hint ; Find the volume of the hall and the mass of the water vapour present. Hence the increase due to the rise in the humidity.

96. A sample of moist air at 27°*C* and 762·75 mm. pressure was found to have a dew-point, when tested equal to 15°*C*. Calculate the mass of 1900 cc. of the air (V.P. of water at 15°*C* = 12·75 mm. of mercury and at 28°*C* = 26·5 mm.).

Hint : Here the air is *moist,* not fully saturated with water vapour. The partial pressure exerted by the vapour in it is given by the pressure of vapour saturated at the dew-point, Refer to Examples 87 and 90.

SOUND

1. Velocity

$$(a) \quad \text{Velocity} = \frac{\text{space}}{\text{time}} \quad or \quad V = \frac{s}{t}.$$

1. The sound of a lightning-thunder was heard 4 sec. after the flash was seen. How far away did the flash occur ? (The velocity of sound in air = 1128 ft./sec.)

Light travels almost instantaneously, while sound travels at 1128 ft./sec. Since the sound required 4 sec. to travel the distance from the clouds to the observer. the distance must be ($1128 \times 4 =$) 4512 ft. $\therefore s = vt$.

2. A man, while standing on a hill, can see a distant railway station and a stationary locomotive. He sees a trail of steam escaping from the locomotive whistle and 2 sec. later, he hears the whistle. The velocity of sound in air at the time is 1148 ft./sec. Determine the distance of the station from the observer.

3. A wood-chopper makes 30 strokes per minute when the speed of sound in air is 1148 ft./sec. The sound of each stroke is heard by a person standing at a distance, as the axe makes the next stroke. How far is the person from the chopper ?

Hint : The time interval between any two consecutive strokes is 2 sec. Find the distance travelled by the sound during the 2 sec. That gives the distance between the two places.

4. Three men A, B and C are standing in a straight line, the distance between A and B being equal to that

between B and C each of which is equal to 3 miles. A gun is fired from a point G. It is heard by A and C at the same instant, but B hears 4·8 sec. before A or C. Determine the position of the gun G, the speed of sound in air being 1100 ft./sec.

Hint : The linee BC, CG and GB form a right angled triangle, the $\angle GBC$ being a right angle. Find the distance $CG-GB$ travelled by the sound during 4·8 sec. Again from $GB^2+BC^2=CG^2$, determine CG or BG.

5. The captain of a warship receives from a light-house the sound of a bell struck under water 5 sec. before he hears the whistle blown above water at the same place. How far is he from the light-house ? The velocity of sound in air is 1096 ft./sec. and that in water 4713 ft./sec.

Hint : Find the time taken by the sound to travel the distance x in water, and that to travel the same distance in air. The difference between the two times is equal to 5 sec, Hence distance x, the distance between the two places.

6. A blow struck upon a steel cable was heard through the cable in 0·3 sec and through the air in 4·5 sec. The temperature was $0^\circ C$. Determine the velocity of the sound in the cable and the distance of the observer from the place where the blow was struck. Velocity of sound in air at $0^\circ C$ is 1088 ft./sec.

7. An observer sets his watch by the report of a signal gun 7 miles away. What allowance should he made on account of the distance of the gun, velocity of sound at the time being 1120 ft./sec.

8. A bullet fired with a velocity of 560 ft./sec. was heard to strike a target 2 sec. after it had left the rifle. Determine the distance of the target from the marksman,

taking the velocity of sound as 1120 ft./sec.

Hint: The time taken by the bullet to travel the distance between the target and the gun *plus* the time taken by the sound to cover the same distance is 2 sec.

9. A stone is dropped into a well 144 ft. deep. What time will elapse before the sound of the splash is heard at the top ? (Velocity of sound in air 1180 ft./sec.

2. Wave-Lengths

$$\text{Velocity of sound} = \text{frequency} \times \text{wave-length}$$

$$v = n \times \lambda$$

where the **frequency** (n) is the number of vibrations per second and the **'wave-length'** (λ) is the distance between any two successive particles which are in the same phase.

(Generally, by *frequency* we understand the number of complete vibrations per second.)

10. A body vibrates in a medium with a frequency of 100 vibrations per second and thereby generates waves each 20 cm. long. Find the velocity of sound in the medium

$$\text{Velocity} = \text{wave-length} \times \text{frequency}$$

$$v = 20 \times 100$$

$$= 2000 \text{ cm./sec.}$$

11. Taking the velocity of sound in air as 1152 ft./sec., find the wave-length of a note of vibration frequency 256 per second.

12. Find the vibration frequency of a note, the wave-length of which is 100 cm. in air and the velocity of sound 330 m./sec.

13. An iron rod, when vibrated, emitted a note of frequency 300 per second. Find the wave-length of the note, if the velocity of sound in iron is 16800 ft./sec.

14. If the frequency of a tuning fork is 240 and the velocity of sound in air 320 m./sec., find how far the first impulse will travel in air when the fork executes 60 vibrations.

15. The edge of a wheel with 32 teeth touches a card as it spins and thereby emits a musical note. If the wheel makes 240 rev. per min., find the frequency of the note.

The number of revolutions per second is (240/60 =) 4.

The card vibrates 32 times per revolution.

$\therefore$ frequency = (32×4 =) 128 vib. per second.

16. A toothed wheel is made to touch a card as it spins. The number of teeth is 30. If the wheel makes 512 rev. per minute, determine the vibration frequency of the note emitted.

17. The disc of a syren contains 60 holes in it. Find the frequency of a note emitted on revolving the disc at the rate of 510 rev. per minute.

As the disc contains 60 holes, the vibrations generated per each revolution of the disc are 60. Hence the frequency of the note is ($60 \times 510/60$) 510 vib. per second.

18. A syren has 28 holes and when sounded gives a note of frequency 560. Find the frequency of rotation of the disc.

3. Vibrating Columns of Air

(i) For a closed resonance tube, of radius r, giving its fundamental note, if the minimum length is l, the wave-length λ of the vibration is given by

$$\lambda = 4 (l + 0{\cdot}6\, r) \qquad \text{(i)}$$

where $0{\cdot}6\, r$ is the end-correction.

(ii) For a tube open at both the ends and giving its fundamental note, if l is its length and r the radius, then the wave-length λ of the note emitted will be given by

$$\lambda = 2\ (\ l+1{\cdot}2\ r\ ) \qquad (\text{ii})$$

Hence the velocity of sound in the gas in the tube is $4\ (l+0{\cdot}6\,r)\ n$ in (i), and $2\ (\ l+1{\cdot}2\ r)\ n$ in (ii).

19. The lengths of air columns vibrating in unison with the two given tuning forks are 22 and 17·6 cm. respectively. If the first fork is G of frequency 384, find that of the other.

Since the frequency n varies inversely as the length of the resonating column of air, the frequency n, of the column of air in unison with the unknown fork is given by

$$n_1 = \frac{n_2 \times l_2}{l_1} \quad \therefore \quad n_1 = \frac{384 \times 22}{17{\cdot}6} = 480.$$

Hence the frequency of the unknown fork is 480 vib./sec.

20. A tuning fork of frequency 256 vibrates in unison with a resonance tube (closed at one end) of length 32·4 cm. What length of the tube will resound in unison with a fork of frequency 384 ?

21. A syren having 16 holes and making 20 rev. per sec. gives a note which is in unison with a resonance tube of length 25 cm. What should be the speed of the syren, if it is to remain in unison with the note emitted by the tube when its length is changed to 30 cm. ?

22. The length of a column of air in a tube closed at one end, which gives the best resonance with a given tuning fork is 34·25 cm. Find the wave-length and the vibration frequency of the note, if the velocity of sound in air is 342·5 m./sec.

Here length $l = 34{\cdot}25$ cm. $= \frac{1}{4}\lambda$, $\therefore$ the required wave-length $\lambda = (4 \times 34.25) = 137$ cm.
Again $v = n\lambda$ $\therefore$ $n = \left(\frac{34250}{137} = \right) 250$. Hence the frequency of the note is 250/ vib./sec.

23. When the velocity of sound in air is 334·4 m./sec., a tube, 20 cm. long and closed at one end, gives the best resonance for a certain tuning fork. What is the vibration frequency of the fork. The diameter of the tube is 3 cm.

24. The shortest air column closed at one end, that gave a resonance with a tuning fork of frequency 320, was 26 cm. long. Find the velocity of sound in air. Diameter of the tube is 4 cm.

25. A tuning fork of frequency 512 was found to be in perfect unison with a resonance tube (closed at one end) 16 cm. long and also when the tube was 49·4 cm. long. Find the velocity of sound in air and also the diameter of the tube.

Hint : The first resonance gives quarter of a wave-length while the second adjustment gives $\frac{3}{4}\lambda$ on adding the end-correction to each. Hence solve for λ and r.

26. A tuning fork is held at the mouth of a resonance tube (diam. = 3 cm.) and resonance occurs when the surface of water is 19·1 cm. below the tuning fork. The frequency of the tuning fork is 434. Find the velocity of sound in air at $0°C.$, the room temperature being $27°C.$ [I.Sc. 1926]

As the diameter is 3 cm. the end-correction is ($0{\cdot}3 \times 3 =$) 0·9 cm. Hence the length of the vibrating air column is ($19{\cdot}1 + 0{\cdot}9 =$) 20 cm. and the wave-length,

$\lambda = (4 \times 20 =)$ 80 cm. $\therefore$ velocity at $27°C.$ is

$v_{27} = (434 \times 80 =)34720$ cm./sec.,and the velocity at $0°C$.
$v_o = v_{27}(1 - \frac{1}{2}\alpha t) = 34720(1 - \frac{1}{2} \times \frac{1}{273} \times 27) = 330{\cdot}3$ m./sec.

27. A closed resonance tube 15 cm. in length responds to a certain tuning fork. the temperature being $31°C$. Find the frequency of the fork. The diameter of the tube is 4 cm. and the velocity of sound in air at $0°C$ is 332 m./sec.

Hint: Find the velocity of sound in air at $31°C$, and then the frequency from the given wave-length.

28. Two resonance tubes when sounded together give 21 beats per every five seconds. If the lengths of the resonating columns of air are 28 in. and 28·5 in. respectively, calculate the frequency of the lower note. Neglect the end-corrections.

The beats given out by the tubes per second $= (21/5) = 4{\cdot}20$. Suppose the frequency of the lower note is n. then that of the higher note will be $n + 4{\cdot}2$. Since $n \propto 1/l$ we have, $(n + /4{\cdot}2)\ 28 = n \times 28{\cdot}5$
$\therefore\ 0{\cdot}5\,n = 117{\cdot}6$. Hence $n = 235{\cdot}2$ vib. per sec.

29. A closed organ pipe gives 8 beats per sec., when sounded with a tuning fork of 256 viv. per sec.—the fork giving the lower note. What change in the length of the pipe should be made to bring it into accord with the fork ?

30. A closed pipe makes 5 beats with a tuning fork with a frequency of 255 per second, the pipe giving the lower sound. What alteration in the length of the pipe will bring it into unison with the fork ? Take the velocity of sound to be 320 *m./sec.* [B.A. 1930]

31. Determine the length of a closed tube full of air which can give the same note as that given by a similar tube 100 cm. long and full of oxygen. Densities of air and oxygen are 1·29 and 1·43 gm./litre respectively.

a. MAGNETISM

1. Coulomb's Law

The force acting between any two magnetic poles m and m' distance d apart in air is mm'/d^2.

A magnetic pole m placed in a magnetic field H is acted upon by a force $F = mH$.

1. Find the force of attraction between two magnetic poles of -40 and 50 units respectively placed 10 cm, apart.

2. Two equal and opposite poles attract each other with a force of 25 dynes, when placed 5 cm. apart. Find the strength of each pole.

3. Two N-poles of magnetic strength 5 and 10 units respectively are placed 10 cm. apart. Find the force of repulsing between them.

4. What is the nature and the magnitude of the force acting between a N-pole of 5 units and a S-pole of 20 units placed 10 cm. apart ?

5. The repulsive force between two magnetic poles is 20 dynes, when they are 5 cm. apart. What will it be if the distance is increased to 6 cm. [B.A. 1924]

The force F between the two poles is mm'/d^2. Hence F, be the required force, and as 20 dynes is the force acting in the first case; we have $\frac{F_1}{20} = \frac{5^2}{6^2}$.

$$\therefore \quad F_1 = \frac{25 \times 20}{36} = 13{\cdot}9 \text{ dynes.}$$

6. Two north-poles repel one another with a force of 4 dynes when they are 2 cm. apart. What will their distance apart be when the force is 1 dyne ? Find also their force of repulsion when they are 3 cm. apart. If the pole-strength of one is two units, find that of the other.

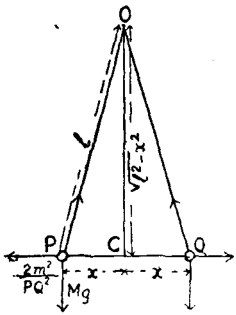
7. The north-pole of a magnetic needle of pole-strength 20 units is acted upon towards one side by a force of 4 gm.wt. Find the strength of the south-pole of a magnet which when kept at a distance of 2 cm. on the opposite side would not allow the needle to deflect from its normal position. Neglect the effect of the other poles of the magnets. $g = 980$ cm./sec^2.

Hint: Convert the force of 4 gm. wt. into dynes. The force is to balance the force of attraction between the two poles.

8. Two magnetic south-poles of strength 40 and 20 units respectively are situated at the corners A and B of a triangle ABC. The sides AC and BC are each 5 cm. If a north-pole of 10 units be placed at C, determine graphically the force acting on the north-pole. $\angle ABC$ being equal to 30°.

Hint: Find by the parallelogram law the resultant of the two forces acting at the point C on the given pole of 10 units. The angle between the two forces is given. Refer to Ex. 3 in static electricity.

9. Two long magnetic needles of the same size are each suspended by two threads 50 cm. in length, attached at either ends, so that the needles keep in a horizontal position and just touch along their entire length. On magnetising these needles equally, they are found to repel each other so that the distance between them is 4 cm. If the weight of each needle is 5 gm., show that th epole-strength of each end of the needle is 39·92 units.



Hint: Here there are two pairs of similar poles. Refer to the example 6 on static electricity for details of the method.

10. A north-pole of 200 units is acted upon by a force of 70 dynes when placed in a uniform magnetic field. Determine the strength of the magnetic field.

The force mH acting on the pole $=$ 70 dynes.

$\therefore$ $200H = 70$; hence $H = 0{\cdot}35$ gauss.*

11. Find the force with which magnetic pole of 100 units is acted upon when placed in a magnetic field of 0·18 gauss.

2. Fields due to Magnets.

(Moments, neutral points, field strengths and their determinations.)

Moment of a magnet $M = ml$ where l is the magnetic length and m the pole-strength of the magnet.

The intensity of a field due to a magnet of moment M and magnetic length $2l$ at a distance d from its midpoint :—

(i) along the axis $= \dfrac{2Md}{(d^2 - l^2)} = \dfrac{2M}{d^2}$ if l is negligible.

(ii) along its equator $= \dfrac{M}{\sqrt{(d^2 + l^2)^2}} = \dfrac{M}{d^2}$,, ,,

12. A magnet 20 cm. long has each of its poles equal to 40 units. Find the moment of the magnet.

13. Calculate the field due to a bar-magnet 20 cm. long and of magnetic moment equal to 1000 units at a

* The unit of magnetic field is called a gauss. It is a field of such a strength that a unit magnetic pole situated in it experiences a force of one dyne. Magnetic field is also expressed in *dynes pe pole or in Oersteds.*

point 20 cm. from each pole. The point lies on the perpendicular bisector of the magnet.

Hint : Find the field at the point due to each of the poles and then the resultant of these two will give the actual field.

14. A 100 unit north pole is placed in a line with a magnetic needle at a distance of 50 cm, from its middle point. If the magnetic length of the needle is 10 cm. and its pole-strength 100 units, determine the force acting on the given pole.

The intensity of the field is given by $F = 2Md/(d^2 - l^2)^2$. Hence mF the total force on the pole

$$= \frac{100 \times 2 \times 1000 \times 50}{(50^2 - 5^2)^2} = 1{\cdot}632 \text{ dynes.}$$

15. A small compass-needle moved in a line along the axis of a bar-magnet (20 cm. long) is found to be in neutral equilibrium at a distance of 20 cm. from its centre. The magnet lies in a uniform field of 0·30 c.g.s. units. Find the moment of the magnet.

16. A short bar-magnet is placed in the magnetic meridian with its north pole pointing south. The neutral point is 24 cm. north of the south pole of the magnet, and upon the production of its axis. Find the intensity of the field at a point on this axis 20 cm. from the south pole and north of it. $H = 0{\cdot}18$. [Pat. Univ.]

Hint : First determine the moment of the magnet. Hence the intensity at a distance of 20 cm due to the magnet alone. The earth's field is opposite in direction. The resultant of the two gives the field required.

17. A bar-magnet of equivalent length 10 cm. and moment 390 pole cm. units is placed horizontally in the magnetic meridian and a small compass needle is found to

stand in neutral equilibrium at a distance of 12 cm. west of the centre of the bar. Calculate the horizontal intensity of the earth's field.

The field due to the magnet at the neutral point is

$$F = \sqrt{\frac{M}{(d^2+l^2)^3}} = \sqrt{\frac{390}{(144+25)^3}} = 0{\cdot}177 \text{ gauss.}$$

But as the compass-needle remains in neutral equilibrium at this point, $H = F$ $\therefore$ $H = 0{\cdot}177$ c. g. s. units (*or* gauss).

18. A nniformly magnetised bar-magnet 10 cm. long and of moment 200 c. g. s. units is placed horizontal with its axis in the magnetic meridian. A small compass-needle placed at a distance of 10 cm. east of the centre of the bar is observed to be in nutural equilibrium. Find the horizontal intensity of the earth's field. [B. Sc. 1924]

19. A magnet is of moment 250 and of length 10 cm. What will be the force due to this magnet at a point (*a*) along the axis 20 cm. away from the north pole, (*b*) along the perpendicular bisector, at the same distance from the north pole? [I. Sc. 1935]

19*a*. Two small bar magnets are fastened together alongside each other with similar poles in contact and are kept along the magnetic meridian. A small compass-needle is found to be in neutral equilibrium if kept on the axial line at a distance of 20 cm. from their centre. But, when one of the magnets is reversed end for end, the needle is required to be moved by 10 cm. towards the magnets to get it in neutral equilibrium. Find the ratio of the pole-strength of the magnets.

20. Two short bar magnets of momet 108 and 192 units are placed along two lines drawn on a table at right angles to each other. Find the field at the point of

intersection of the lines, the centres of the magnets being respectively 30 and 40 cm. from this point . [I.Sc. 1934]

3. Couples acting on magnets

When a magnet of moment M is deflected through θ in a uniform field of intensity H, restoring couple

$$C = MH \sin \theta \qquad \text{(i)}$$

If a magnet of moment M is deflected from its normal position through an angle θ in a uniform field of intensity H, then the work done against the magnetic forces

$$= 2\, HM \sin^2 \tfrac{1}{2}\, \theta$$

21. Determine the moment of a couple required to deflect a magnet 10 cm. long through 60° from the magnetic meridian. The pole-strength of the magnet is 10 units. $H = 0{\cdot}2$ gauss.

The force $m\,H$ acting on each pole $= (10 \times 0{\cdot}2 =)2$ dynes. The perpendicular distance between the forces on the poles $= 10 \sin 60°$. Hence the moment of the couple$=(2\times 10 \sin 60° =) 17{\cdot}34$ dyne-cm.

22. If a couple-acting on a magnet 12 cm. long having a pole-strength of 7 units placed at an angle of 30° to the uniform magnetic field is 21 dyne-cm., determine the strength of the field.

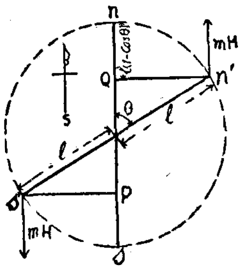
23. N-pole of a long magnet ($m = 480$) is place at a perpendicular distance of 40 cm. from the centre of a horizontally suspended small magnetic needle of length 4 cm. and pole-strength 100 units. Find the couple acting on the small needle neglecting the effert of the S-pole of the long magnet and the earth's field.

24. A horizontally suspended magnet is deflected first through 18° and then through 38°10′ from its position

of rest. Compare the torques (*i. e.* the turning moments of the forces) acting on the magnet in different positions.

***25.** A magnet of pole-strength 30 units and length 20 cm. makes an angle of 30° with the magnetic meridian. Calculate the turning couple which tends to bring the magnet back to the magnetic meridian. Also calculate the work done in rotating the magnet through 60° from the magnetic meridian. ($H = 0{\cdot}36$ dyne per unit pole.)

[B. A. 1927]



(i) Refer to Example 21 for the first part. (ii) the force F acting on each pole $=$ ($30 \times 0{\cdot}36 =$) $10{\cdot}8$ dynes. When the magnet is deflected, the poles move along a circle; but this is equivalent to some distance perpendicular to the field and some distance against the field. This latter is $l(1 - \cos\theta)$ where l is half the magnetic length of the magnet and θ the angle of deflection. Work is required to move each pole along this distance against the uniform field. Hence the total work Fs done on the two poles $= [\,2 \times 10{\cdot}8 \times 10\;(\,1 - 0{\cdot}5\,)\,] = 108$ ergs. *Or* using the relation (ii), $W = (2 \times 30 \times 20 \times 0{\cdot}36 \times \frac{1}{4} =)$ 108 ergs.

***26.** A small pivoted compass needle ($M = 200$ pole-**cm.**) free to move in a horizontal plane lies at 30° to the east of the magnetic meridian in a field of 0·2 gauss. Calculate the change in its potential energy if it is deflected through 30° more.

4. Coulomb's Torsion Balance.

27. A bar-magnet is suspended in a uniform mag-

netic field by means of a thin silver wire. The wire is twisted through an angle of 100° in order to deflect the magnet through 30°. With what twist in the wire will the magnet be deflected through 45° ? [S. E. 1912]

The deflecting couple is proportional to the torsion in the wire, and the resorting couple is $MH \sin \theta$. The torsion is (100 – 30 =) 70° for a deflection of 30°. If θ, be the torsion for a deflection of 45°, we heve,

$$\frac{\theta}{70^\circ} = \frac{MH \sin 45^\circ}{MH \sin 30^\circ} \quad \therefore \ \theta = 99^\circ,$$

and the total twist is (99 + 45 =) 144°.

Note:— If the deflection is small *i. e.* not greater than 14° $MH \sin \theta$ may be taken as $MH\,\theta$ without much error; then θ should be in radians.

28. A bar-magnet suspended in a uniform magnetic field by a fine wire, having no twist to start with, is turned through an angle of 30° by turning the torsion head through 120°. What additional angle must the torsion head be turned through in order to treble the angle between the magnet and the field ?

29. A bar-magnet is suspended by a wire so as to hang horizontally. The top of the wire is twisted through 210° and the magnet is deflected through 30° from the meridian. The same magnet is removed, remagnetised and restored to the same position and the top of the wire has to be turned through 390° to deflect the bar as much as before. Compare the moments of the bar before and after the remagnetisation.

5. Tangent Magnetometers.

For equilibrium of the needle due to a magnet of moment M in a field H in the

(i) End-on position *or* A position of Gauss

$$\frac{M}{H}=\frac{(d^2-l^2)}{2}\tan\theta \text{ or } \frac{d^2\tan\theta}{2} \text{ if } l \text{ is negligible.}$$

(ii) Broad-side-on position *or B* position of Gauss

$$\frac{M}{H}\sqrt{(d^2+l^2)^3}\tan\theta \text{ or } d^3\tan\theta, \text{ if } l \text{ is negligible;}$$

where d is the distance of the axis of rotation of the needle from the centre of the magnet and l half the magnetic length of the magnet.

30. Find the moment of a small magnet which when placed at a point 40 cm. due east of a compass needle with its axis along the centre of the needle produces a deflection of 8°. ($H=0{\cdot}18$ c. g. s. units.)

If F be the field due to the magnet and H the earth's horizontal field acting on the needle of moment M' the couple due to the magnet is $M'F\cos\theta$, and that due to H is $M'H\sin\theta$. When the needle is in equilibrium $M'F\cos\theta=M'H\sin\theta$ $\therefore$ $F=H\tan\theta$, Since the magnet is very small, neglecting its length, we have $F=2M/d^3$ where M is the moment of the magnet. Hence $M=\frac{1}{2}d^3 H\tan\theta=(\frac{1}{2}\times 40^3\times 0{\cdot}18\times 0{\cdot}1405=)$ 809·2 pole.cms.

31. A magnet is placed "end on" with reference to a magnetometer with its centre 30 cm. away. The mean deflection of the magnetometer needle is 45°. What is the moment of the magnet? ($H=0{\cdot}37$ c.g.s. units) [B.A. 1925]

32. A deflection magnetometer is adjusted with its arms due East and West. A magnet 20 cm. long is placed on one of the arms along its length with a centre at a distance of 20 cm. from the needle and the deflection produced is 30°. Find the moment of the magnet (1) neglecting its length, (2) considering its length as not a negligible factor ($H=0{\cdot}2$ dyne per pole.)

33. A short bar-magnet is placed with its axis along the magnetic East and West direction. A small compass needle when placed 70 cm. away from the centre of the magnet along a line perpendicular to its axis points 30° away from the magnetic meridian. If the value of H at the place is 0·32 c.g.s. units, find the moment of the bar-magnet. [I.Sc. 1928]

The couples acting on the needle are $M'H \sin \theta$ and $M'F \cos \theta$, where H and F are the fields acting on the needle; and M' and θ its moment and deflection respectively. But $F = M/d^2$ neglecting length l of the magnet.

$\therefore \; M = d^2 H \tan \theta = (\; 70^3 \times 0{\cdot}32 \times 0{\cdot}5774 =)\;$ 63370 pole-cm.

34. A tangent magnetometer is so adjusted that its arms are in the magnetic meridian. A bar-magnet with its axis East-West is put on one of the arms with its centre at a distance of 20 cm. from the needle and the deflection is 11°. Determine the moment of the magnet neglecting its length ($H = 0{\cdot}3$ gauss.)

35. A short bar-magnet placed at a distance of 40 cm. from the centre of a magnetometer-needle produces in the end-on position a deflection of $13^\circ\ 30'$. At what distance must it be placed in the broad-side-on position in order to produce a deflection of $6^\circ\ 1'$? Determine also the moment of the magnet. ($H = 0{\cdot}36$ gauss.)

36. In a deflection magnetometer experiment two small magnets are placed perpendicular to the magnetic meridian 30 cm. north and 40 cm. south respectively from the needle. Compare their moments, if the needle keeps undeflected.

37. A bar-magnet A (10 cm. long) kept at a distance of 40 cm. in the end-on position, deflected the magnetometer needle by 31° ; while the other magnet B (12 cm. long) is required to be placed 50 cm. away from the needle to get the same deflection. Compare the moments of A and B.

38. A small bar-magnet 50 cm. to the west to a compass needle deflects it through 45° at a place A ; while at a place B, the same magnet deflects it through 31° under the similar conditions. Compare the Earth's horizontal field at A and B.

39. Two short bar-magnets lie 20 cm. apart along a line at right angles to the magnetic meridian. A compass needle placed midway between them is deflected through 30° from the meridian. The deflection rises to 60° when one of the magnets is removed. Compare its moment with that of the magnet which remains. [L.U.]

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b. STATIC ELECTRICITY

1. Coulomb's Law

The force F exerted between two point charges e_1 and e_2 distant d apart in air is e_1e_2/d^2.

1. Two small spheres are placed 10 cm. apart in air. If one of them carries a charge of 27 units of positive electricity and the force of attraction exerted between the two is 8·1 dynes, find the charge on the other.

Taking a negative sign for the force of attraction F

The force $F = -8{\cdot}1$ dynes. Hence $\dfrac{27 \times e}{10^2} = -8{\cdot}1$

$$\therefore \quad e = -\frac{8{\cdot}1 \times 10 \times 10}{27} = -30 \text{ E. S. Units.}$$

2. Calculate the strength of a field at a point equidistant from each of the two equal and opposite charges of 100 units, when the distance between them is 10 cm. Find the direction, of the field also.

Hint: The magnitudes of the forces are equal and they act in the same direction, *viz.*, towards the negative charge.

3. Two small spheres, charged with 100 E. S. U. each, are placed respectively at the corners B and C of an isosceles triangle ABC, where $AB = AC = 50$ cm., and $\angle BAC = 60^\circ$. Find graphically or otherwise the strength and the direction of the electric field at A.

4. Calculate the force on a charge of -10 units placed at the intersection of the diagonals of a square at the corners of which are placed point charges of 2, $-$ 10,

20 and 10 units respectively. Each side of the square measures 10 cm.

Hint: Find the intensity of the electric field at the point considered. Hence the required force on the given charge.

5. Charges of 100 units each are placed at the two opposite corners A and C of a square $ABCD$ the side of which is 10 cm. If a charge of -50 units is placed at B, find the direction and the magnitude of the force which would act on a charge of -10 units placed at D.

6. Two small spheres, each of mass one decigramme, are suspended from a point by threads each 50 cm. long. They are equally charged and repel each other so that the distance between them is 20 cm. What is the charge on each? [I.Sc. 1928]

Let P and Q (Refer to the figure on example 9 in magnetism) represent the given spheres suspended from a common point O. Let e be the charge on each of the spheres, then the force of repulsion between the two will be e^2/PQ^2 acting along the horizontal, Each sphere is now in equilibrium under the action of three forces *viz.*, (i) the force of repulsion acting horizontally, (ii) the weight of the sphere acting vertically downwards and (iii) the tension in the string. Hence taking the moments of the forces about the point of suspension O, we get

$$\frac{e^2}{PQ^2} \times OC = mg\frac{PQ}{2} \quad \therefore \frac{e^2}{20^2}\sqrt{50^2-10^2} = 0{\cdot}1 \times 980 \times \frac{20}{2}$$

Hence on simplifying we get $e = 89{\cdot}45$ E.S.U.

7. Two spherical pith balls, each of mass 0·2 gm. and each charged with e units of positive electricity, are suspended by light insulated threads of length 50 cm. from

the same point. In the quilibrium position on each thread makes an angle of 5° with the vertical. Prove that e is equal to 36·2 units.

8. Two small balls, weighing 0.5gm. each, are suspended vertically from two points 3 cm. apart in the same horizontal plane, by means of silk fibres, each one decimetre long. On imparting equal charges of opposite kind to the spheres, they are found to approach one another until the distance between the two is reduced to 2 cm. What is the change on each ?

Hint: Consider the forces keeping the ball in equilibrium in the displaced position. Hence take the moments of these forces about the point of suspension of the fibre.

9. Find the charge on a small body if its weight appears to increase by 0·02 gm. when it is placed 10 cm. vertically over another small body carrying a charge of 100 E. S. U.

****10.** The bob of a simple pendulum is a sphere of mass 10 gm, and it is suspended by a silk-theread, so that the length of the pendulum is 70cm. Vertically beneath it is placed a second sphere carrying an electric charge of 400 E. S. U. Determine the period, if the bob is given a charge of −350, E. S. U. and is kept 10 cm. apart in air from the other sphere. The amplitude of vibration is so small that the attaractive force may be taken acting along the vertical.

For a simple pendulum $T=2\pi\sqrt{l/g}$ where g is the actual acceleration in the vertically downward direction In the present case two downward forces

** Example 10 and 11 may be omitted. They are not in the syllabus.

act on the bob, one due to its weight and the other due to the attraction between the two electric charges. If the acceleration on the bob due to the electric force be α, the total downward force will be $g+\alpha$ per unit mass. The force of attraction between the charges is ($350 \times 400/10^2 =$) 1400 dynes, hence $\alpha = \left(\frac{1400}{10}\right)$ 140 cm./sec.^{2} $\therefore \alpha = F/m$

$$\therefore \text{ period } T = 2\pi\sqrt{\frac{l}{g+\alpha}} = 2\pi\sqrt{\frac{70}{980+140}} = 1{\cdot}571 \text{ sec.}$$

**11. The bob of a simple pendulum adjusted to beat seconds is a sphere of 10 gm. Vertically beneath it, is kept a second sphere carrying a positive charge. When the pendulum bob is negatively electrified, the period is 1·0 sec. Determine the attractive force between the two due to the electric charges. (Assume the attractive force to act in a vertical direction.)

Hint: Find the total downward acceleration. Hence the acceleration due to the electric force. The bob is affected by this attraction. Mass of the bob is known. Calculate the required force.

12. Two equal spheres of water carry equal charges of the same kind of electricity. Find the change in the surface density of electrification that will take place when both combine to form a large sphere without loss of any charge.

Hint: Find the surface area when both the spheres coalesce. The total charge gets uniformly spread over this area.

2. Electric Potential

Electrostatic potential at any point is equal to the work, in ergs, required in order to convey a unit positive

charge from an infinite distance upto that point.

The potential V due to an insolated charge Q at a point distant d from it is Q/d.

Work is to be considered as *negative* when the charge is moving *down* the grade of potential and *positive* when *up* the grade of potential *i.e.* work is to be considered positive when it is done upon the charge and by an external agency. Note that 'potential' is a scalar quantity.

13. A small isolated conductor carries an electaic charge of 100 E. S. U. Find the potential at a point distant 20 cm. from it.

The required potential is $Q/d = (100/20 =)$ 5 units.

14. Two small conductors 40 cm, apart have each, a positive charge of 10 and 50 units respectively. Calculate (i) the force per unit positive charge, (ii) the potential, midway between them.

(i) The force due to 10 units is $10/20^2$. and that due to 50 units is $50/20^2$., $\therefore$ the resultant force $= [(50-10)/400 =]$ 0·1 dyne and is directed towards 10 unit charge. (why?); (ii) The potential $V_1 = (\frac{10}{20} =)$ 0·5. The potential $V_2 = (\frac{50}{20} =)$ 2·5. The resultant potential due to both the charges $= (2{\cdot}5 + 0{\cdot}5 =)$ 3 units.

15. At each of the corners of an equilateral triangle, the side of which is 8 cm., is placed a charge of 100 units of electricity. Find the potential at the point of intersection of its medians.

16. A square $ABCD$ of side-length 8 cm. has electric charges of magnitudes 16, 34 and 24 *e.s.u.* placed at the three corners B, C and D respectively. What is the resultant potential at A? [B.A. 1929]

17. If 1000 ergs of work is done upon a charge of 10 units in moving it from a point at a potential of 50

units to a point at a potential V, find the value of V.

Hint: Since the work is done *upon* the charge, it is moved to a point at a higher potential. Hence V is greater than 50.

18. Find the energy spent when a charge of 25 electrostatic units is taken from a place of potential 50 to a place of potential -15. [I. Sc. 1932]

19. Two equal and opposite charges $+e$ and $-e$ are at a distance apart 5×10^{-9} cm. In order to remove one of them away from the other to infinity, work has to be performed against them amounting to $4{\cdot}55 \times 10^{-11}$ ergs. What is the numerical value of e in *e. s.* units ? [I.Sc. 1935]

20. Charges of 10 units each are placed at the corners of a square the side of which is 4 cm. Determine the potential at the point of intersection of the diagonals. How much energy is expended in carrying a charge of 10 units from the mid-point of a side to the point of intersection of the diagonals ?

Hint: Determine the potential at the two points considered. The difference between these two potentials gives the amount of work required to move a unit charge from one point at a lower potential to the other at a higher one.

***21**. A particle of mass 9×10^{-28} gm. carrying a charge of $4{\cdot}77 \times 10^{-10}$ E. S. U. moves freely in a *vacuum* between two plates between which a constant P. D. of 100 E. S. U. is maintained. Find the velocity acquired by the particle.

Hint: Kinetic energy acquired by the mass is due to the work done in moving from one plate to the other. Obtain the velocity by equating the two energies.

3. Capacities of conductors.

The capacity of a conductor $C = QV$ .... (i)
The capacity of a spherical conductor of radius r cm $= r$ (ii)
The resultant capacity of a number of conductors of capacities C_1, C_2, C_3 ...... C_n, when joined by a *fine* wire is $C_1 + C_2 + \ldots\ldots + C_n$ .... (iii)

22. A spherical conductor of radius 10 cm. is imparted a charge of 200 units. Find the potential at the surface.

The charge act as if it were concentrated at the centre of the sphere. Hence the potential is

$$Q/d = (\ 200/10 = \ )\ 20 \text{ units.}$$

23. A spherical conductor has a charge of 1000 units and is at a potential of 100 units. What is its radius?

24. A hollow spherical conductor of 10 cm. radius carries a charge of 100 units of positive electricity. Find the potential (1) at the surface of the conductor, (2) inside it and (3) at points at distance of 30 and 40 cm. from its surface. If the conductor is connected by a *long thin* wire with another spherical conductor of 10 cm diameter find the charge and the potential of each sphere.

(1) $\because$ $C = r = 10$ cm. the potential at the surface $V = (\ Q/C = \ )\ 100/10 = 10$ units.

(2) No charge resides inside a closed hollow conductor; the potential inside is the same everywhere, and is equal to that on the surface. Hence $V = 10$ units.

(3) i. At the point distant 30 cm.

$$V = \left[\frac{100}{30+10} = \right] \quad 2{\cdot}5 \text{ units.}$$

ii. At the point distant 40 cm.

$$V = \left[\frac{100}{40+10} = \right] \quad 2 \text{ units.}$$

(4) The capacity of the new sphere $C_2 = r_2 = 5$ cm., hence the resultant capacity when the two are joined $C = (10+5=)$ 15 cm. $\therefore$ the common potential $= \frac{100}{15}$ and the charge on the first sphere is $C_1 V = (\frac{10 \times 100}{15} =$ 66·66 units and on the other $(\frac{100 \times 5}{15} =)$ 33·33 units.

25. The metal spheres of radii 6 and 12 cm. electrified with 400 and 320 positive units respectively, are placed a long way apart and then connected by a very fine insulated wire. Find the final potential of the sphere. [I. Sc. 1934]

26. A metal sphere of 10 cm. radius carrying a positive charge of 100 units brought into contact with another metal sphere of 5 cm. radius carrying a negative charge of 50 units. Find the charge on and the potential of each sphere after contact. [I. Sc. 1925]

Hint : The resultant charge is shared by the spheres in the ratio of their capacities. The final potential of both is the same.

27. Two insulated spheres of radii 10 and 5 cm. respectively have their centres 20 cm. apart. The potential of B is 5 and the work done in carrying 4 units of electricity from B to A is 20 ergs. Calculate the potential of A and the charges on A and B. What will be their potential when they are connected by a wire ? [I. Sc. 1930]

28. An insulated sphere of 10 cm. radius is charged to a potential of 100 units, it is then made to share its charge with a Leyden jar. If the resultant potential is found to be 80 units, find the capacity of the Leyden jar.

Hint: First find the total charge, then the resultant capacity. Hence determine the capacity of the jar.

4. Capacities of Condensers

Capacity of a sphercial air condenser having the radius of the inner sphere r_1 and that of the inner surface of the outer earth-connected sphere r_2 is $\frac{r_1 r_2}{r_2 - r_1}$.

The capacity C of a parallel plate condenser $= A/4\pi d$.

The intensity F of a field between the plate of a parallel plate condenser with air as dielectric $= 4\pi\sigma$.

29. A spherical air condenser, the coatings of which have radii 4 cm. and 5 cm. respectively, is charged to a potential of 20 units, the outer coating beings in contact with the earth. Find the capacity of the condenser and the charge on it. State what would happen if air is replaced by sulphur.

The capacity of the outer coating is 5 cm, and that of the inner one 4 cm; therefore, the capacity of the condenser is $\left[\frac{4 \times 5}{(5-4)} = \right]$ 20 cm. The magnitude of the charge on it is $CV = [\,20 \times 20\,] = 400$ units.

Capacity of the condenser increases when air is replaced by sulphur because the potential of the charged conductor falls.

30. Determine the ratio of numerical values of the capacities of a sphere of 10cm. radius when it is surrounded concentrically by another hollow sphere, once of 12 cm. radius and another time of infinite radius. [B. A. 1929]

31. The plates of a parallel plate condenser are 220 sq. cm. each and are separated by air. Find the capacity of the condenser, when the distance between the plates is 2·5 mm.

The capacity is $\frac{A}{4\pi d} = \frac{220 \times 7}{4 \times 22 \times 0{\cdot}25} = 70$ cm.

32. At what distance the plate (diameter = 4 cm.) of an air condenser should be placed so that it can have the same capacity as that of a sphere carrying a charge of 100 units and having a potential 20 units ?

Hint: First find the capacity of the sphere. This is equal to the capacity of the plate condenser. Hence determine the required distance.

33. A condenser consists of the two circular parallel plates of 4 cm. diameter, and one of them is earthed. At what distance must the other plate be kept so that the condenser possesses the same capacity as that of a conducting sphere of 100 cm. diameter immersed entirely in air upto infinity ? [I. Sc. 1936]

Hint: Capacity of the plate condenser has to be made equal to that of the sphere of radius 50 cm.

34. The plates A and B of a parallel plate condenser of capacity 20 units are 3 mm. apart. A is insulated and B is earthed. When A is charged positively the electric intensity at any point between A and B is 4 units and acts normally to the plates. Calculate the charge on A. If the electric intensity remains unaltered when A is moved towards B by 1 mm., calculate the capacity of the new arrangement. [I Sc. 1926]

Here the capacity $A/4\pi d = 20 \quad \therefore \; A = 20 \times 4\pi \times 0{\cdot}3 = 24\pi$ sp. cm. But the electric intensity $F = 4$. It also must be equal to $4\pi\sigma \; \therefore$ the surface density $\sigma = 1/\pi$.

$\therefore$ total charge $Q = A\sigma = 24$ units.

When the plates are taken nearer, the capacity is

$$\frac{A}{4\pi d} = \frac{24\pi}{4\pi \times 0{\cdot}2} = 30 \text{ cm.}$$

5. Specific Inductive Capacity.

Specific Inductive Capacity (S.I.C.) of a substance

$$K = \frac{\text{capacity of a condenser with the sub. as dielectric}}{\text{capacity of the same condenser with air as the ,,}}$$

35. An air condenser has a capacity of 50 e. st. units. On changing the dielectric, the same condenser shows the capacity of *400 e. st. units.* What is the dielectric constant (*or* S. I. C.) of this new substance ?

36. In the example 31, if the air is replaced by mica of specific inductive capacity 6·6, find the capacity of the condenser.

37. Two plates each 20 cm. square kept at a distance of 0·2 mm. form a parallel plate condenser. Find the intensity of the field between the two plates and the capacity of the arrangement (1) when the space between the plates contains air and (2) when the space is filled up wiht paraffin wax of S. I. C. 2·5. The insulated plate is given a charge of *1000* units.

Condensers in series and in parallel

The resultant capacity C_r of a number of condensers, having their capacities C_1, C_2, C_3, ... respectively is given by

(i) $C_r = C_1 + C_2 + \ldots$ when joined in parallel.

(ii) $\frac{1}{C_r} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ when joined in series.

(*or* in cascade.)

38. 3 condensers of capacities 4, 5 and 10 cm. respectively are connected (*a*) in parallel (*b*) in series. Calculate the resultant capacity in each case.

(*a*) The resultant capacity $C_1 = (4+6+10=)$ 23 cm.

(*b*) The resultant capacity C_r is given by

$$\frac{1}{C_r} = \frac{1}{4} + \frac{1}{6} + \frac{1}{10} \therefore C_r = \frac{60}{31} \text{ cm.}$$

39. What is meant by charging Leyden jars, 'in cascade'? Three Leyden jars whose capacities are 2, 4 and 6 cm. respectively are joined in series. What is the final capacity of the arrangement?

40. 4 Leyden jars, each having a capacity of 100 units, are joined in parallel. Find the capacity of the arrangement.

41. 3 condensers of capacities 10,20 and 30 cm. respectively are joined in parellel and then charged to a potential difference of 40 units. Find the total charge given and the amount of charge shared by each.

42. Two condensers whose capacities are 10 and 5 E. St. Units respectively are connected in series and the combination is joined to a source of potential difference of 300 E. St. Units. Find the charge and the potential difference for each condenser.

CURRENT ELECTRICITY

1. Forces acting on a Conductor Carrying a current in a magnetic field

The mechanical force acting on a conductor, of length l, placed perpendicular to a magnetic field H and carrying a current C is lCH.

If the current is measured in amperes, the force will be one-tenth of the above, as ten amperes of a current equal one absolute unit of it.

Generally all the measurements in electricity are taken in *C. G. S.* system. 1 oersted = 1 gauss *i.e.* 1 dyne per pole.

1. Find the mechanical force acting on a straight conductor 20 cm. long placed at right angles to a magnetic field of 50 oersteds. The current passing through the conductor is 10 amperes.

The mechanical force acting on the conductor

$$lCH = (20 \times \tfrac{10}{10} \times 50 =)\ 1000 \text{ dynes.}$$

2. An electric power line is two kilometres long and is at right angles to the magnetic meridian. Find the force acting upon the wire if it carries a current of 91·8 amperes and $H = 0{\cdot}3$ c.g.s. units. [I.Sc 1946]

3. Find the magnitude and direction of the force acting on a straight conductor 50 cm. long placed at right angles to the earth's magnetic field H of 0·30 oersted. The current enters the conductor at the upper end and is of strength 2 amperes.

Hint: Apply the left-hand rule. The fore *finger* shows the direction of the fild, the *middle finger* the dire-

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*In all the formulæ in this chapter symbol C is used for a current in absolute units I for a current in practical units.

ction of the current while the *thumb* shows the motion giving the direction of force.

4. Determine the strength of a magnetic field in which a conductor, 10 cm. long, carrying a current of 5 amperes when placed inclined at 30° to it, is acted upon by a force of 1·75 dynes.

Hint: Here the component of the field perpendi cular to the length of the conductor is effective. The component parallel to the direction of the current does not exert any force on the conductor. Hence calculate the force due to the other component.

5. A wire, 100 cm. long, was lying at right angles to a magnetic field of strength 0·2 oersted. On passing a current of 10 amperes the wire moved at right angles across the field through a distance of 10 cm. Find the work done.

6. A rectangular coil carrying a current of 2 amperes, when freely suspended in a magnetic meridian, sets itself in a vertical plane inclined at 30° to the magnetic meridian. The vertical sides of coil are each 30 cm. long and the horizontal ones 20 cm. Determine the couple acting on the coil. ($H = 0·2$ oersted.)

> The mechanical force acting on each of the vertical wires is $lCH = (30 \times 0·2 \times 0·2 =)$ 1·2 dynes. Since the coil makes 30° with H, the normal to the coil is at 60°. to H. Hence the perpendicular distance between the two forces trying to turn the coil is 20 sin 60°
> $\therefore$ couple $l\,C\,H\,b \sin 60° = (1·2 \times 20 \times 0·865 =)$
> 20·78 dyne-cm.

7. Find the couple acting on a rectangular coil of length 30 cm. and width 20 cm. situated in a vertical plane making an angle of 60° with a field of 0·2 oersted.

There are 30 turns of the wire in the coil and a current of 0·1 ampere flows in it.

Hint: Each turn behaves as an independent coil. The total effect is that of all the turns added together.

8. A coil of 80 turns, each embracing an area of 25 sq. cm., carries a current of 2 amperes. Find the value of the couple acting on the coil, when it is held in a vertical plane inclined at 30° to the magnetic field of strength 20 units.

9. A circular coil of 10 turns and of radius 10 cm. carries a current of 2·6 amperes. What couple will be required to keep the coil parallel to the magnetic meridian? ($H=0·3$ gauss.)

10. A square coil, having each side 50 cm. and carrying a current of 10 amperes, is held with its plane parallel to the magnetic field of strenth 0·4 c.g.s. units. Calculate the work done when the coil moves to the position of stable equilibrium. (I. Sc. 1929 ;

Since the coil moves to the position of stable equilibrium, the deflection of the coil from the field is 90°- and the total number of lines of force (N) cut by the coil is $AH=(50^2\times 0·4=)1000$.

Hence the work done NC = $(1000\times \frac{10}{10}=)$ 1000 ergs. *Or* slove thus:—Each of the vertical conductors are acted upon by a force of $(50\times 1\times 0·4=)$ 20 dynes. The point of application of each force moves through a distance of $(\frac{50}{2}=)$ 25 cm. Hence the work done by each of the forces in taking the coil in the position perpendicular to the field is $(25\times 20=)$ 500 ergs. The total work done, therefore, is $(500\times 2=)$ 1000 ergs. (Compare the example 25 on magnetism.)

11. A vertical circular ring (radius $=6$ cm.) carrying a current of 4 amperes is in equilibrium in the earth's magnetic field, when perpendicular to the magnetic meridian. What energy shall have to be spent in turning the ring round the vertical axis until its plane coincides with the meridian ? ($H=0{\cdot}4$ oersted.)

12. Calculate the work done in turning the coil in example 10 from the position inclined at 30° to that of 60° to the magnetic field.

Hint : In the 30° position, the lines of force enclosed by the area are $A\,H \sin 30°$, while in the 60° position, they are $H\,A \sin 60°$. Hence the work done is the iscrease in the lines of force threading through the area multiplied by the current.

2. FIELD DUE TO A CIRCULAR CONDUCOTR

The magnetic intensity F due to a current C in a circular coil of n turns each of average radius r is

(i) $\dfrac{2\pi nC}{r}$, at the centre of the coil.

and (ii) $\dfrac{2\pi\, n\, Cr^2}{\sqrt{(r^2+d^2)^3}}$ at the point on the axis at a distance d.

If the current is in amperes the intensity will be one-tenth of the above.

13. A current of 3·5 absolute units passes through a circular coil of radius 10 cm. What is the magnetic intensity at the centre of the coil ?

The current is. 3·5 ab. units and the number of turns in the coil unity, hence, from relation

$$F=\frac{2\pi nC}{r}, \text{we have, } F=\frac{2\pi\times 3{\cdot}5}{10}=2{\cdot}2 \text{ oersteds.}$$

14. What current must pass through a circular conductor of mean radius 5 cm., in order to have a force of $\frac{22}{7}$ dynes exerted on a magnetic pole of 10 units placed at the centre of the coil ?

Hint: The magnitude of the force acting on the 10-unit pole is given. Find the force per unit pole. That is the magnetic intensity at the centre of the coil. Hence determine the current C or I.

15. Calculate the strength of a magnetic field at the centre of a circular coil of 20 turns and 20 cm. diameter due to a current of 10 amperes flowing in it. Determine the force which would act upon a magnetic pole of 10 units placed at the centre of it.

The magnetic field at the centre of it,

$$\frac{2\pi nC}{r} = \frac{2 \times 3{\cdot}142 \times 20 \times 10}{10 \times 10} = 12{\cdot}57 \text{ gauss.}$$

Hence the force acting on the given pole of 10 units $= (10 \times 12{\cdot}57 =)$ 125·7 dynes.

16. What is the intensity of the magnetic field at the centre of the coil of a tangent galvanometer of 20 turns of wire, and of radius 11 cm., when a current of 0·42 ampere is passed through the coil.

17. The force exerted on a 100-unit pole placed at the centre of a coil of wire of 100-turns and a mean diameter 20 cm. is 6284 dynes. Determine in amperes the current flowing through it.

18. A field due to a current of 20 amperes in a circular loop of wire repels a 5-unit pole at the centre of the loop with a force of 20 dynes. What must be the radius of the loop ? [I. Sc. 1922]

19. How many turns are there in a coil (of wire) of negligible cross-section and of radius 15·4 cm which produces a field of 6 c. g. s. units at its centre when a current of 3 amps. flows through it ? [I. Sc. 1945.]

20. Find the strength of a magnetic field at a point 20 cm. away on the axis of coil of wire of 20 turns and mean radius 10 cm., when a current of 10 amperes passes through it.

We have here $r=10$ cm. $n=20$, $d=20$ cm. hence using the relation (ii) we get

$$F=\frac{2\times 3{\cdot}142\times 20\times 1\times 10^2}{\sqrt{(10^2+20^2)^3}}=1{\cdot}124 \text{ oersteds.}$$

21. The force on a magnetic pole of 100 units placed at a point 40 cm. away on the axis of a loop of wire of 10 turns and radius 30 cm. is 22·623 dynes when a current of electricity is passed through the wire. Calculate in amperes the strength of the current.

22. Two similar coils of wire, having a radius of 4 cm. and 10 turns have a common axis and are 6 cm. apart. If a current of 1 ampere flows through each of the coils in the same direction, find the strength of the magnetic field (*a*) at the centre of either coil and (b) at a point on their common axis midway between them.

[I. Sc. 1928.]

Hint : Find F_1 and F_2 due to each coil at the points considered. The sum of these two fields at the point will give the required field. They are in the same direction. Use relation (ii)

23. Calculate the strength of the magnetic field in the above example taking the same current to flow through each of the coils in opposite directions.

Hint: The fields at the points considered to opposse each other. The resultant gives the required field.

24. The coil of a tangent galvanometer is of 10 turns of an average radius 2π cm. It is kept in a plane perpendicular to the magnetic meridian. A small magnetic needle, freely suspended at the centre of the coil, makes 20 oscillations in 10 seconds. On passing a current through the coils, the needle takes the same time to make 30 oscillations, without changing the direction of its poles. Find the currrent strength. $H = 0{\cdot}30$ gauss

Let H′ be the field due to the current I amperes then in the first case the field is H while in the other it is $H'+H$. Hence as the frequency of oscillation varies as the square root of the field in which the magnet vibrates

$$\frac{n^2{}_1}{n_2} = \frac{H'+H}{H}, \quad \therefore \frac{H'+0{\cdot}30}{0{\cdot}30} \therefore 2{\cdot}70 = 4H+1{\cdot}20.$$

$\therefore \quad H' = \left(\frac{1\cdot5}{5} = \right) 3{\cdot}375$ gauss.

Again, this field H' at the centre of the coil due to the current I is $\dfrac{2\pi \times 10 \times I}{10 \times 2\pi}$, but $H = \dfrac{1{\cdot}50}{4}$, $\therefore \; I = 0{\cdot}375$ ampere.

24*a*. Find the frequency of vibrations in the above example, if the current is 0·5 ampere, (1) when the direction of the needle remains the same, (2) when the direction gets reversed. What will be the effect on the needle and on the period, if the current is changed to 0·25 amp, (1) once in the original direction, (2) other time with the direction reversed ?

25 A circular coil of 15 turns of wire of mean radius 6π cm. was held vertically in a plane parallel to the magnetic meridian. A short horizontal magnetic needle

freely suspended at the centre of the coil performed 10 oscillations in 16 seconds when no current passed through the coil. On passing a current of one ampere the needle required 10 seconds to perform the same number of oscillations. Determine the value of H. Also state the period of oscillation on reversing the direction of the current in the coil.

Hint : The needle is influenced by the resultant of the two fields acting mutually at right angles. Find this resultant and also F due to the current. Hence the value of H.

3. TANGENT GALVANOMETERS

In a tangent galvanometer, when a current C of absolute units (*or* of I amperes) flows through the coil of n turns each of mean radius r which is kept parallel to a magnetic field H.

$$C = \frac{r\,H}{2\pi n}\tan\theta \text{ or } I = \frac{10\,r\,H}{2\pi n}\tan\theta$$

$$= \frac{H}{G}\tan\theta, \text{ where } G = \frac{2\pi n}{r} \text{ the constant of the galvanometer.}$$

$$= k\tan\theta, \text{ where } k = H/G, \text{ the reduction factor of the galvanometer.}$$

Quantity of electricity is measured in **coulombs** and is equal to the product of the current in amperes and the time in seconds during which the current is passing in a conductor *i.e*, $Q = It$.

26. Find the reduction factor and also the constant of a tangent galvanometer having 100 turns of wire of mean diameter 20 cm. ($H = 0{\cdot}1885$ gauss.)

27. A current of 10 amperes produces a deflection of 45° in a tangent galvanometer. What is the value of the

current which will produce a deflection fo 30° in the same galvanometer ? [C. U.]

Hint : Current is proportional to the tangent of deflection.

28. Electricity is passing through a tangent galvanometer at the rate of 15·285 coulombs per minute and the observed mean deflection of the needle is 27°. Determine the reduction factor and the constant of the galvanometer.

The current $C = \frac{15 \cdot 285}{60 \times 10}$ absolute units.

Also $C = k \tan 27^\circ \therefore k = \dfrac{15 \cdot 285}{10 \times 60 \times 0 \cdot 5095} = 0 \cdot 05$ units.

The constant $G = \dfrac{H}{k} \therefore G = \dfrac{0 \cdot 3}{0 \cdot 05} = 6.$

29. The radius of the coil of a tangent galvanometer is 20 cm. and the coil has 5 turns of wire. The horizontal component of the earth's magnetic field is 0·314 gauss. How many coulombs of electricity will pass through the galvanometer per minute when the deflection is 45° ?

30. A tangent galvanometer of radius 20 cm. and 100 turns of wire showed a deflection of 45° when its coil is placed parallel to the earth's field. An ammeter in the same circuit showed a reading of 100 milliamps. Find the value of H. [I. Sc. 1931]

31. A vertical coil of mean radius 10 cm. and consisting of 20 turns is placed in the magnetic meridian. A deflection of 33°, when a current of 0·25 ampere flows through the coil. Calculate the value of H.

In the case of a tangent galvanometer, current

$$I = \frac{10rH}{2\pi n} \tan\theta. \quad H = \frac{0 \cdot 25 \times 2\pi \times 20 \times 1 \cdot 54}{10 \times 10} = 0 \cdot 484 \text{ gauss.}$$

32. A tangent galvanometer has 100 turns of wire of radius 10 cm. It gives a deflection of 45° with a current of 1 ampere when its coil is parallel to the field H. Find the value of H. Assuming that it is used to measure currents giving a deflection between 30° and 70°, find the range of currents in amperes for which it can be used.

33. Calculate the intensity of the magnetic field at the centre of the coil of tangent galvanometer of 19 turns of wire and 15 cm. radius, when a current of 0·45 ampere is passed through the coil. If the earth's field H is 0·358 c.g.s. units, find the deflection of the galvanometer needle.

[B.A. 1927.]

34. A circular coil of 10 turns and mean radius 20 cm. is held vertically in the magnetic meridian. A small magnetic needle, freely suspended at a distance of 10 cm. along the axis from the centre of the coil, shows a deflection of 35° when a current is allowed to pass through the coil. Find the current. ($H = 0{\cdot}2$ gauss.)

Hint: First find F in terms of the current, then use the tangent formula which will give the strength of the current.

35. The vertical plane of the coil of a tangent galvanometer with 10 turns is inclined to the magnetic meridian by 30° to the West. A current of 0·4 ampere is passed through the galvanometer coil. If the needle deflects by 30° to the East, calculate the value of the horizontal component of the earth's magnetic field. (Radius of the coil $= 4\pi$ cm.) [I. Sc. 1927]

The force F due to the coil acts at right angles to its plane. The needle is controlled by the effect of the two fields F and H, and makes an angle of 30° with H. Hence,

$$MH \sin 30^\circ = MF \cos 60^\circ$$

$$\therefore \quad MH \times 0{\cdot}5 = M \times 2\pi \times 10 \times 0{\cdot}04 \times 0{\cdot}05/4\pi$$

$$\therefore \quad H = 0{\cdot}2 \text{ gauss.}$$

36. A circular coil of 5 turns has a radius equal to 10 cm. and is placed in a vertical plane making an angle of 30° with the magnetic meridian. Calculate the value of the horizontal component of the earth's magnetic field, if a small magnetic needle placed at the centre of the coil is deflected through a right angle when a current of two amperes circulates through the coil. (B. A. 1925.)

Hint: The component of F in the magnetic meridian just neutralizes H. The needle is at rest under the influence of the other component of F perpendicular to the meridian.

37. The coil of a tangent galvanometer is kept in the magnetic meridian and a current of 0·2 ampere is allowed to flow through it. The coil is then rotated through 30° about the vertical axis so that the galvanometer needle and the coil both come in the same plane. What is the horizontal intensity of the earth's magnetic field, if the coil has 10 turns of a wire of mean radius 6·284 cm ?

Here the deflecting field due to the current in the coil is perpendicular to the axis of the needle, and hence the equation of equilibrium is

$$F = H\sin\theta. \text{ But } F = \frac{2\pi nC}{r} = \frac{2\pi \times 10 \times 0{\cdot}2}{6{\cdot}284 \times 10} = 0{\cdot}2 \text{ gauss;}$$

$$\therefore \quad H = \frac{0{\cdot}2}{\sin 30^\circ} = 0{\cdot}4 \text{ gauss.}$$

38. The coil of a certain tangent galvanometer is capable of rotation about a vertical axis. When a current of 0·5 amp. is passed through the coil fixed in the magnetic

meridian, the deflection observed is 45°. Another current is then passed through the galvanometer and the coil rotated and finally kept in the plane of the needle. Find the value of the current if the deflection observed is 30°.

[I. Sc. 1933]

38a. A sine galvanometer has a coil of 10 turns of an average diameter of 4π cm. On passing a current through it, the coil has to be rotated about a vertical axis through 30° from the magnetic meridian in order to have the needle in the same plane as the coil. Calculate the current. ($H = 0{\cdot}2$ gauss.)

4. OHM'S LAW

Electrict Circuits

Ohm's Law states that when two points are taken on a conductor, the ratio of the difference of potential V between these points to the current C flowing through it is constant. This constant is called the resistance of the said conductor between the points considered. Hence,

(i) $$\frac{P.D.}{\text{current}} = \text{resistance}, \frac{V}{C} = R, or \frac{\text{Volts}}{\text{Amperes}} = \text{Ohms}.$$

(ii) when a cell is working on a circuit,

$$I = \frac{V}{R_{ex}} = \frac{E - V}{R_i} = \frac{E}{R_{ex} + R_i}$$

where I = the current in *amperes*

V = P.D. (in *volts*) at the terminals of the cell.

E = E. M. F. (in *volts*) of the cell.

R_{ex} = external resistance (in *ohms*) in the circuit.

R_i = internal resistance (in *ohms*) of the cell.

(iii) When a galvanometer of resistance g is shunted by a resistance s, the current in the galvanometer $Cg = \dfrac{sC}{s+g}$, where C is the current in the main circuit.

39. Two flash-light electric bulbs are marked $2{\cdot}5V$, $0{\cdot}3A$, and $3{\cdot}5V$, $0{\cdot}3A$ respectively. If V and A stand for volts and amperes respectively, determine the resistance of the bulbs when lit up.

From Ohm's Law, resistance is given by $\dfrac{P.\,D.}{\text{current}}$

$\therefore$ $R = (\frac{2\cdot5}{0\cdot3} =)\ 8\frac{1}{3}$ ohms, the resistance of one bulb. Similarly the resistance of the other bulb is $(\frac{3\cdot5}{0\cdot3} =)$ $11\frac{2}{3}$ ohms.

40. What is the resistance of an electric heater (when *hot,) if a current of 0·5 ampere passes* through it, when joined to a 110 volt circuit ?

41. What current flows through an electric heater of 200 ohms resistance placed across a 250-volt circuit ?

42. An electric current pass through a coil of resistance 15 ohms. If the $P.D.$ between the terminals of the resistance is 1·5 volts, determine the current.

43. Three resistances of 10, 15 and 25 ohms respectively are joined in series, and a current is sent through them at a voltage of 12 volts. Determine the current.

Here the resistance R of the circuit is

$$R = (10+15+25=)\ 50 \text{ ohms.}$$

Hence the current is $\dfrac{V}{R} = \dfrac{12}{50} = 0{\cdot}24$ ampere.

44. A maximum current that a given electric lamp of 95 ohms resistance can draw from an electric supply at 110 volts is 0·25 amp. What resistance will have to be put in series in the circuit ?

45. The $P.\ D.$ at the ends of an 18 ohm resistance coil in an electric circuit is 3 volts. What is the $P.\ D.$ across the terminals of a 6 ohm resistance in the same circuit ?

Hint: The same current passes through both the resistances. Use the relation $P.\ D. \propto$ resistance.

46. A 220-volt circuit contains two motors in series having their resistances 100 and 200 ohms respectively. Find the fall of potential through each. Calculate the current also.

47. A 220-volt current passes through 10 lamps in series, each lamp having a resistance of 55 ohms. Determine the current passing through the lamps and the fall of potential through each. Calculate the $P.D.$ necessary to send the same current through each of the lamps when arranged in parallel.

The total resistance of the lamps in series is

$$R = (10 \times 55 =)\ 550 \text{ ohms}$$

Hence the current I in the lamps $= \frac{220}{550} = 0{\cdot}4$ ampere.

The fall of potential through each lamp is

$$IR = (0{\cdot}4 \times 55 =)\ 22 \text{ volts.}$$

This also shows that in the parallel arrangement $P{\cdot}D{\cdot}$ required by each lamp is 22 volts. *or*

When the lamps are in parallel, the total effective lamp resistance R is given by $\frac{1}{R} = \frac{10}{55}$ $\therefore\ R = 5{\cdot}50$ ohms.

Since each lamp takes 0·4 ampere, the total current required is $(0{\cdot}4 \times 10 =)$ 4 amperes.

$\therefore\ P.D. = (4 \times 5{\cdot}5 =)$ 22 volts.

48. A uniform wire of resistance 12 ohms is bent into a circle and two points at a quarter of the circumference apart are connected to a battery whose resistance is 1 ohm and e.m.f. 3 volts. Find the current in the different parts of the circuit. (I.Sc. 1940)

49. A flash-light battery is run for some time through an external resitance of 8 ohms. The current given by one battery is 0·47 amp. in the beginning but falls to 0·27 amp. during the time, and the voltage gets reduced to 3·0 volts from 4·3 volts. Find the initial and final values of the resistance of the cell.

50. 10 electric lamps, each having a resistance of 22 ohms (when hot), are arranged in parallel and a total current of 2·5 amperes is required to bring the lamps to their proper candle-powers. Determine the E. M. F. required.

51. An electric current of 5 amperes is divided into three branches, the lengths of the wires in the three branches being proportional to 1: 2: 3. Find the current through each, assuming the wires to be of the same material and cross-section.

52. An ammeter joined in series in an electric circuit containing some unknown resistance and a cell of e. m. f. 1·5 volts showed a current of 0·15 ampere. Determine the value of the unknown resistance, if the internal resistance of the cell is 2 ohms.

Neglecting the resistance of the ammeter, we have a resistance of $x+2$ ohms in the circuit, taking x as the unknown resistance to be determiend.

Hence $x+2=(\frac{1\cdot5}{0\cdot15}=)\ 10 \quad \therefore \quad x=8$ ohms.

53. A 10·5 volt battery of resistance 0·5 ohm sends a current through an external resistance. If the current

passing through the resistance is 0·5 ampere, find the external resistance.

54. When an 8 volt battery, of resistance one ohm, is connected through a lamp, the P. D. between the poles falls to 7 volts. Find the resistance of the lamp.

Let r be the resistance of the lamp, then from Ohm's law we have $\frac{8}{1+r} = \frac{7}{r} = I \quad \therefore\ 7+7r=8r$ *or* $r=$ 7 ohms.

∴ The resistance of the lamp is 7 ohms.

55. A cell, having an E. M. F. of 1·2 volts, sends a current of 0·1 ampere, when its terminals are joined through a resistance of 10 ohms. Determine the resistance of the cell.

56. The E. M. F. of a certain battery is 12 volts and the current sent by it through an external resistance of 6 ohms is 1·2 amperes. Calculate the resistance of the battery.

57. The E. M. F. of a standard battery is 21 volts. When it sends a current of 1 ampere through an external resistance, the P. D. at the terminals falls to 19 volts. Determine the resistance of the battery.

Here a P. D. of (21 – 19 =) 2 volts is driving the current in the battery. The current in the circuit is 1 ampere. Hence from Ohm's law, the resistance of the battery = ($\frac{2}{1}$ =) 2 ohms.

58. A circuit containing a storage cell and resistance has a current of 10 amperes passing through it. The P. D. between the terminals of the cell is 1·8 volts. When the circuit is broken the P. D. increases to 1·98 volts. Find the resistance of the cell.

59. The E. M. F. of a battery is 20 volts. When the poles are connected by a copper wire, a current of 2

amperes flows in the circuit and the P. D. between the battery terminals fall through 4 volts. Find the resistance of the wire and that of the battery.

60. A dynamo, the internal resistance of which is 10 ohms and the E. M. F. 63·75 volts, sends a current through an external resistance of 10 ohms in series with a set of 3 resistances 5, 10 and 2 ohms respectively in parallel. What is the total resistance in the circuit, and the current flowing through the dynamo ?

The effective resistance R of 5, 10 and 2 ohms in parallel is given by

$$\frac{1}{R}=\frac{1}{5}+\frac{1}{10}+\frac{1}{2}=\frac{16}{20} \quad \therefore R= 1{\cdot}25 \text{ ohms.}$$

Hence the total resistance in the circuit is

$(1{\cdot}25+10+10=)$ 21·25 ohms.

$\therefore$ the current $= (\frac{63{\cdot}75}{21{\cdot}25})$ 3 amperes.

61. A single dynamo operates 1000 similar lamps at 110 volts. The lamps are arranged in parallel and each of them takes a current of 0·5 ampere. Find the total current supplied by the dynamo and the resistance of each lamp.

Since each lamp takes a current of 0·5 ampere and as they are in parallel, the total current taken by the lamps is. $(1000\times0{\cdot}5=)$ 500 amperes. $\therefore$ the current supplied by the dynamo is 500 amperes.

If r be the resistance of each lamp, then from Ohm's law, we have $r=(\frac{110}{0{\cdot}15}=)$ 0·220 ohms.

62. 110 tantalum lamps (0·34 A and 110 V) are connected in 55 rows of 2 each. The dynamo and the line

have resistance 0·6! and 1·3 ohms respectively. What must be the e.m.f. of the dynamo? [B. A. 1929]

63. 10 lamps, each of 55 ohms resistance, are arranged in series. The leads and the generator have a resistance of 5 ohms and 1 ohm respectively. Calculate the E.M.F. and the P.D. of the generator if a current of 1 ampere passes through the circuit.

Hint: The E.M.F. and the P.D. of the generator (*i.e.* dynamo) play the same role as the E.M.F. and the P.D. of a battery.

64. A current from a cell with a tangent galvanometer and a resistance coil in series with it, gave a deflection of 30°. On replacing this cell by another, the deflection was 45°. Compare the E.M.F.s of the cells, if the total resistance in the circuit remained unaltered.

65. A cell *A* is allowed to send a current through a tangent galvanometer ($g=90$ ohms) and a resistance coil of 30 ohms, and the deflection is noted. On replacing the cell *A* by *B*, the same deflection of the galvanometer is obtained by decreasing the risistance of 30 ohms to 10 ohms. Neglecting the resistance of the cells, find the E.M.F. of the cell *A*, if that of the cell *B* is 1·5 volts.

Let E_A be the E.M.F. of cell *A*, and E_B that of *B*,

then $\dfrac{E_A}{30+90}=k\tan\theta=\dfrac{E_B}{10+90}$ $\therefore$ $\dfrac{E_A}{E_B}=\dfrac{120}{100}$

$$\therefore \quad E_A = 1{\cdot}2\ E_B = (\ 1{\cdot}2\times 1{\cdot}5=)\ 1{\cdot}80 \text{ volts.}$$

66. A cell, of internal resistance 2 ohms, is conected to a tangent galvanometer the resistance of which is 4 ohms and the deflection is found to be 58·2°; when some resistance was introduced into the circuit the

deflection was reduced to 37·2°. Find the resistance introduced and the E.M.F. of the cell. [I. Sc. 1924]

$C = k \tan\theta$ in absolute units. If practical units are used, $I = 10\,k \tan\theta$. Hence if R is the resistance introduced,

by ohms' law, $I_1 = \dfrac{E}{2+4} = 10\,k \tan 58{\cdot}2^\circ$ (i)

and $I_2 = \dfrac{E}{2+R+4} = 10\,k \tan 37{\cdot}2^\circ$ (ii)

Dividing (i) by (ii) $\dfrac{6+R}{6} = \dfrac{1{\cdot}6128}{0{\cdot}759}$ and $R = 6{\cdot}750$ ohms

Using (i), $E = (\,6 \times 10 \times k \times 2{\cdot}6128 =\,)\ 96{\cdot}7\ k$ volts.

67. A tangent galvanometer, having a coil of 50 turns and a mean diameter of 40 cms., is connected in a circuit carrying an electric current. The total resistance of the circuit is 3·142 ohms. If the deflection given by the current is 45°, determine the P. D. between the terminals of the battery sending the current, The resistance of the battery is negligible. ($H = 0{\cdot}2$ oersted) .

Hint : Calculate the reduction factor first; then find the current and the P.D. The current as given by the galvanometer will be in absolute units.

68. A tangent galvanometer is placed in series with a cell of E.M.F. 1·5 volts and the internal resistance of 2 ohms. and the deflection observed is 31°. Find the resistance of the galvanometer, if the radius of the coil be 10 cm. and the number of turns in it be 30. ($H = 0{\cdot}3142$ gauss) .

69. A circuit, consisting of a galvanometer and a resistance of 20 ohms in series, showed a deflection of 45°

in the galvanometer, On changing the resistance to 50 ohms, the deflection was 35°. Calculate the resistance of the galvanometer, neglecting the resistance of the battrey.

70. A daniell cell is connected in series with a tangent galvanometer of one ohm resistance and a box of resistance coils. When a resistance of 2 ohms is taken out of the box, the deflection of the galvanometer is 60° and when the resistance in the box is increased to 20 ohms. the deflection falls to 30°. Find the resistance of the cell.

[C. U.]

71. A circuit consists of a battery, a resistance of 10 ohms and a tangent galvanometer of 20 ohms. resistance joined in series. The deflection noted is 45°. On introducing an extra coil in the circuit, the deflection falls to 31°. Required the resistance of the unknown coil. Neglect the battery resistance.

MISCELLANEOUS EXAMPLES

General Physics

1. An airplane flying at a speed of 140mi/hr. encounters a wind of 40 mi/hr. from the south-west. In what direction must the plane head so as to travel eastward ? Also find its velocity relative to the ground.

2. A simple pendulum suspended from a roof of a railway carriage is in a vertical position. Find its inclination to the vertical when the train is just accelerating at the rate of 3 ft./ sec^2.

3. A man throws balls up into the air one after the other, starting each one as the previous one reaches the top of its flight. Find the maximum height to which each ball will rise, if he throws two per second. Find also, the total time of flight of each ball.

4. A boy with outstretched arms drops a marble ball from his hand. When the ball falls through one foot, he lowers his hand with a uniform acceleration and catches the ball the moment it is 2 ft. below its starting point. With what acceleration does the boy lower his hand?

5. A body weighing 2 lb. rests on an outstretched hand. Find the force with which the body pushes on the hand when the hand is moved upwards with the acceleration of 3 ft./sec^2. With what acceleration the hand should move, if the pressure were to be double the weight of the body?

6. A weight of 20 lbs. rests on a horizontal plate ascending (a) with a constant velocity of one foot per

second, (b) with a velocity constantly increasing at the rate of one foot per second per second. Find in each case the force on the plate. ($g=32$ ft./sec^2.) [I.Sc. 1938 Math]

7. A man weighing 160 lbs. is accelerated upward by an elevator at 5 ft./sec^2. What will be the force exerted by the floor of the elevator on the man ? State the direction of the force also.

8. A sled leaves the top of a hill at a speed of 200 cm./sec. and acquires a uniform acceleration of 80 cm./sec^2. while in motion. Find the distance travelled during the 4th second.

9. A gun fires a shell with a muzzle velocity of 490 in. per sec, at an elevation of 37°. (a) Find the maximnm height it would reach and the time required for it. (b) What will be its horizontal range ?

10. A pendulum is suspended from the ceiling of a tramcar that is moving on a horizontal track with a uniformly accelerated motion of 8 ft. /sec^2. Find the inclination of the pendulum to the vertical. With what force will a 140-lb. passenger press against the side of his seat ?

11. A small paper pellet weighing 0·7 gm is found to fall 420 cm. in one second after starting from rest. Find the average resistance of the air on the pellet. ($g=980$ cm./sec^2.) [I. Sc. 1929]

12. A cannon weighing 1000 kg. free to move on a horizontal plane, fires a shell of 10 kg. at an elevation of 45°. If the maximum, height reached by the shell is 3200 metres. what would be the velocity of recoil of the cannon ?

13. Two forces acting on a particle are at right angles and are balanced by a third force making an angle of 150°

with one of them. The greater of the two forces is 3 lb.wt. What are the values of the other two ? [I. Sc. 1938 Math.]

14. A uniform bar of length 7 ft. 6 in. and weight 17 lbs. rests on a horizontal table with one end projected 2 ft. 6 in. over the edge. Find the greatest weight that can be attached to its end without making the bar topple over. [I. Sc. 1938 Math.]

15. A rope hangs down a smooth pulley and a man weighing 60 kgm. slides down the portion of the rope on one side of the pulley with a uniform acceleration of 140 cm. / sec^2. Find with what uniform acceleration a man weighing 50 kgm. must slide down (or pull himself up along) the other portion of the rope so that the *rope* may remain at rest. (g = 980 cm./sec^2.) [I. Sc. 1921]

16. A diver releases a bubble of air the volume of which gets doubled on reaching the surface. Find the depth of the diver. The barometer reads 76 cm. of mercury at the time.

17. In an air pump the volume of the receiver is 4·485 times the volume of the barrel. How many strokes are necessary to reduce the pressure in the receiver to $\frac{1}{5}$th of the original pressure ? [I. Sc. 1939]

Heat

18. The volume of a thermometer bulb is 0·80 cc. and is filled with mercury. If the mercury rises in the stem *at the rate of* 1 *mm. per* 1°*C* rise in temperature, determine the cross-section of the stem.

19. A piece of glass which weighs 90 gm. in air is found to weigh 49·6 gm. in a certain liquid at 12°*C*. At 97°*C* its apparent weight in the same liquid is 51·9 gm. If the coefficient of cubical expansion of glass is 0·000024,

find the coefficient of absolute expansion of the liquid.
[I. Sc. 1934]

20. Air is pumped in an automobile tyre under a gauge-pressure of 35 lb/sq. in at 27°*C*. After a trip in the hot sun, the temperature was found to be 57°*C*. *Find the* total pressure exerted by the confined air when hot.

21. The pressure in a constant volume gas thermometer immersed in melting ice is 60 cm. and when immersed in boiling water it is 80 cm. The thermometer is now placed in another liquid whcih is boiling and the pressure recorded is 90 cm. Find the boiling point of the liquid. [I. Sc. 1939]

Hint: Determine α from the first two readings. Hence using this α find the temperature required *or* solve graphically.

22. An air-tight bladder-holding 2 litres of air at N.T.P. is immersed in sea-water to a depth of 13·6 m. where the temperature is 4°*C*. *What is the volume of the* gas in the bladder at the depth? Sp. gr. of sea-water 1·02·

23. A man requires to breathe the same mass of air per minute at the top of a mountain as at the foot. The respective pressures and temperatures are 75 cm. and 27°*C*. at the foot and 50 cm. and 7°*C*. at the top. If he always fills his lungs to the same extent, how many times will he breathe per minute at the top? He requires to breathe 20 times per minute at the foot.

24. A gas ring burning 30 cu. ft. of gas per hour is used to heat water. If the calorific power of the gas is 500 B. Th. U. per cubic foot and 60 per cent of the heat gets to the water, how long will it take (*a*) to raise the temperature of 5 points of water from 62°*F* to the boiling point, (*b*) for the kettle to boil dry after the boiling

commences ? (Pint of water weighs 20 ozs. Latent heat of vaporisation of water is 972 B. Th. U. per lb.)

[I.Sc. 1934]

25. The specific heat of ice is 0·5. What quantity of heat will be required to change 10·2 gm. of ice at $-76°C.$ to water at $14°C.$? [I.Sc. 1934]

26. A lump of ice at a temperature of $-20°C.$ was dropped in a calorimeter of thermal capacity 5 calories containing 95 gm. of water at $34·7°C$, The final temperature was found to be $9·7°C.$ and the weight of the water 120 gm. Compute the specific heat of ice.

27. 5 gm. of steam at $100°C.$ are introduced in a vessel containing a mixture of 216 gm. of water and 10 gm. of ice in thermal equilibrium. At the end, the temperature of the vessel and its contents is $10°C.$ What is the water equivalent of the vessel if the latent heat of steam is 540 calories and that of ice 80 calories ? [I.Sc. 1938]

28. Steam at $10°C.$ passes into a radiator in a room containing 100 kg. of air at $10°C.$ and the water resulting from the condensation of the steam leaves the radiator at a temperature of $36°C.$ How many grams of steam are used in warming the air in the room to $25°C.$? The specific heat of air is 0·24 and the latent heat of steam 536 cal./gm.

29. A calorimeter of thermal capacity 10 calories contains 390 gm. of a liquid (sp. ht. 0·47) at $26°C.$ and is heated by a bunsen burner at a constant rate. The liquid attains its boiling point ($156°C.$) in 5 min. How long will it take to evaporate the whole of the liquid ? Latent heat of vaporisation of the liquid is 69 calories per gm.

30. 100 gm. of steam at $200°C.$ are passed into a calorimeter containing 400 gm. of water and 400 gm. of ice at

$0°C$. What should be the final temperature of the mixture? Water-equivalent of the calorimeter is 17 gm. The specific heat and the latent heat of steam are respectively 0·467 and 540 calories per gm.

31. A mass of air is at a temperature of $100°C$. Its dew point is $48°C$. If ths saturation pressure of aqueous vapousr at $48°C$ is 8·35 cm., find the relative humidity of the air. [I.Sc. 1939]

32. Calculate the difference in temperature between the two sides of an iron plate 2 cm. thick when heat is transmitted at the rate of 600 kg. calories per sq. metre per minute. (The thermal conductivity of iron 0·2). [I.Sc. 1937]

Light

33. Calculate the diameter of the real image of the moon formed by the telescope objcetive of focal length 60 ft. Distance of the moon from the earth is 240000 miles and its diameter 2000 miles.

34. Water in a beaker is 8 cm. deep. A pin is moved vertically upwards above the surface of the water till its image in the water appeared to coincide with a minute piece of a metal lying at the bottom just below. The distance of the pin above the surface of water is found to be 6 cm. Determine the refractive index of water.

35. A fish, 12 in. in length, is swimming directly towards the plane glass-front of an aquarium. How long will it appear to a person looking through the glass at the moment, when it is 2 ft. away from it ? ($\mu = \frac{4}{3}$).

36. An observer looks vertically down a pool of still water 5 ft. deep. His eye is 5 ft. above the surface. He observes the image of his eye and also the image of a

stone lying at the bottom just below. What are the positions of the images, the refractive index of water being $\frac{4}{3}$? Also draw a neat diagram. [I.Sc. 1932]

37. A short-sighted person wishes to see a design at the bottom of a narrow glass jar. He gradually adds water to the jar and looks at the design with his eye at the mouth of the jar. When the water rises to 8 cm. in the jar he sees distinctly the design as well as the image of his eye in water. If the refrative index of water is $\frac{4}{3}$, find (*a*) the height of the jar, (*b*) the distance of distinct vision of the person. [I.Sc. 1930]

38. A horizontal pencil of sodium light is transmitted through an aperture in a vertical screen and is allowed to fall normally on the vertical face of a right-angled prism. The angle of the prism is 30° and the refractive index is 1·54. Calculate the deviation of the ray. [B.A. 1931]

39. *ABC* is a prism with refractive index 1·5. The angle *A* is a right angle and *B* is an agnle of 60°. A ray is incident normally on the face *AB*. Trace the path of the ray until it emerges from the prism and find the deviation produced (Sin 48° 40′ = 0·75.) [I.Sc. 1927]

40. It is required to throw an image magnified 29 times on a screen placed at a distance of 6 metres from the object. Find the focal length and the nature of the lens required. [I.Sc. 1939]

41. Find the position and nature of the image of an object kept at a distance of 12 cm. from a convex lens a focal length 8 cm.

By how much distance will the image be shifted if a parallel plate of glass 10 cm. thick is interposed (1)

between the image and the lens, (2) between the object and the lens ? (μ for glass = 1·5).

Hint: In (1) apparent thickness of the plate is $\frac{10}{1.5}$ cm, hence the reduction in v by ($10 - \frac{10}{1.5} =$) $\frac{10}{3}$ cm., while in (2), u gets reduced to (12 – 3·3) cm. and the consequent increase in v.

42. A beam of light converges to a point P. A convex lens of focal length 20 cm. is interposed so as to intercept the converging pencil of rays. If the distance of the lens from P is 30 cm., find the distance of a point from the lens where the rays meet. What would be the effect of replacing the convex lens by a concave one of 30 cm. focal length and of the same material ?

43. An optician prescribes lenses of – 4 dioptres to a person for reading a book comfortably when held at a distance of 30 cm. from the eye. (1) State the nature of the defect. (2) Find the distance of the near point when not wearing the spectacles.

44. The near and far points of a man who has lost the power of accommodation of his eyes are 75 cm. and 300 cm. respectively. What kind of spectacles will you prescribe to enable him to see comfortably, (*a*) a country side, (*b*) a book held at a distance of 25 cm. from the eye ? How will you combine these lenses in one ?

45. Two electric lights are kept 2·5 m. apart. If their candle-powers are in the ratio of 9 : 4, where must a grease spot be placed between them so that it cannot be seen ?

46. In an experiment with a Rumford's photometer, a lamp with a dirty chimney, when placed at a distance of 10 cm. from the screen balanced a candle. On cleaning

the chimney, the lamp had to be moved away from the screen by 2 cm. to balance the same candle which was not moved during the experiment. Calculate the percentage of light which was being absorbed by the dirty chminey.

Magnetism

47. Find the distance of the neutral points from the centre of a small bar magnet of moment 200 pole-cms. when placed in a field of 0·2 gauss (*a*) with its south-pole pointing North (*b*) north-pole pointing South.

48. A magnet of pole-strength 150 units and the magnetic length 20 cm. is laid on a table along the magnetic meridian with its S-pole pointing towards the North. Find the resultant magnetic field at a point 20 cm. north of the magnet from its centre. ($H=\frac{2}{3}$).

49. A small magnet is placed in the magnetic meridian with its south-pole pointing north and on the prolongation of its axis at a distance of 20 cm. from its mid point, a small compass-needle is placed. When the magnet is rotated through 30° about the pivot of the needle, the needle is found to have kept itself at right angles to the axis of the magnet. Calculate the magnetic moment of the magnet, if $H=0{\cdot}2$ c.g.s. units. [I.Sc. 1931]

50. Two small magnets *C* and *D* have moments of 500 and 600 units respectively. The centre of *C* is situated on the perpendicular bisector of *D* at a distance of 50 cm. from the centre of *D*. Find the maximum value of the couple on *C* due to *D*. [I. Sc. 1939]

51. Two magnets *A* and *B* are placed with their axes at right angle to each other and their middle points coincident. Compare the magnetic moments of these two magnets, if the magnetic field on the axis of *A* and at

a distance from the magnets is at an angle of 45° to the axial line.

52. A small bar magnet kept at a distance of 35 cm. from the needle of a tangent magnetometer in the end-on position caused the needle to deflect by 34°, while on replacing this magnet by another a deflection of 52° was obtained. Compare the moments of the two.

Static Electricity

53. Two small spherical pith balls weighing 180 mg. each were suspended from the same point by threads 130 cm. long. Some electrostatic charge was imparted to them when they were in contact. On getting charged, they were found to repel each other to a distance of 100. cm. Find the charge on each.

54. Find the charge in a small body if its weight appears to increase by 0·02 gm. wt. when it is placed 10 cm. vertically over another small body carrying a charge of 100 e. st. units.

55. A 10 mg. pith ball carrying a charge of 8 micro-coulombs, is placed in an electric field of 10 dynes per coulomb. Determine its initial acceleration under the effect of the field.

56. A pith ball of mass 10mg. is charged with 4 E.S.U. and is kept 50 cm. just directly above a charge of -5 E.S.U. Find the total force acting on the ball. What will be its initial resultant acceleration downwards ?

57. If 4 joules of work are spent in moveing a charge of 20 E.S.U. from infinity to a certain point, find the increase in the potential of the point.

58. Two conducting spheres of raddii 50 and 10 cm. respectively are joined by a long thin conducting wire

and a total charge of 1200 units of positive electricity is imparted to one of them. Find the surface density on each sphere.

59. Two spherical conductors have equal charges, but volume of one is twice that of the other. If connected by means of a thin and metallic wire, will a current pass through it and from which conductor to which ? Find the final charge on each conductor. [I Sc. 1938]

60. An insulated spherical conductor A is charged to a potential of 10 *e. st. u.* and the electric field near its surface is then 0·1 *e.st u.* A is then connected to another uncharged spherical conductor, B placed at a distance. As a result the electric field near A falls to $\frac{3}{4}$th of its original value. Calculate (1) radii of A and B, (2) and charge which flows from A to B, (3) the final potential of A and B. [I.Sc. 1939]

61. Find the force acting on a particle having a mass of 9×10^{-28} gm. and carrying a charge of $1{\cdot}9 \times 10^{-20}$ *e. st. u*. while it is passing in vacuum between two plates 5 mm, apart, the *P.D.* between the plates being 100 *e. st. u.*

62. A leyden jar has a diameter of 20 cm. and is made of glass 2·5 mm., in thickness. If the height of the cylindrical coating is 20 cm., find its capacity (S. I. C. of glass is 6).

63. If the potential of a sphere is raised from 10 to 15 units by the addition of 100 units of a charge, find the radius of the sphere.

ANSWERS

Units and Measurements

1. *Fundamental units:*—
gm., ft., cm.
Derived units :—
pdl., erg., watt.
hr. ft.-lb., H. p.

2. *Absolute units*:—
ergy, dyne-cm.,
Poundal, watt.
Gravitional units:—pound,
H.P., g-cm.,ft-lb.

3. *Scalar*:—
volume density
work, speed,
mass, potential,
Vectors:— force
surface tension,
acceleration,
wt. momentum.

4.

	M.	L.	T.
(ii)	0	0	0
(iii)	1	1	– 1
(iv)	0	0	– 1
(v)	1	– 3	0
(vi)	1	1	– 1

5.

(i)	1	1	– 2
(ii)	1	2	– 3
(iii)	1	0	– 2
(iv)	0	3	0
(v)	0	0	0

6.

(i)	0	1	– 2
(ii)	1	– 1	– 2
(iii)	1	– 1	– 2
(iv)	1	1	– 2

8. 742 watts
12. $n \propto \sqrt{T/ML}$
13. 0·01 cm.

14. 0·05 mm.
15. $40M = 25V$
16. $24M = 25V$
17. $59M = 60V$ 1 minute.
18. 0·01°
19. 0 312 in. 0·002 in.
20. 29·928 in.
21. 3·78 cm.
23. 1·05 cm.
24. +0·05 4mm 2·23 mm.
25. 3·60 mm.
27. 66·69 cm.
28. 21·396 cm.
29. 13·68.
30. 0·25 mm.
31. (*a*) 1% (*b*) 2%

PROPERTIES OF MATTER

2. 8·43 g./cc. **3.** 2 cc. **4.** 225000 gallons. **6.** 6 cm. **7.** (*a*) 646·8 gm. (*b*) 950·4 gm. **8.** 22·025 gm. **10.** 2 mm. **11.** 0·001181 g/cc. **12.** 0·818. **14.** 11·28 g./cc. **15.** 2·68

17. 2 : 1 **18.** 0·869 **19.** 1 g./cc. **20.** 1·16 g./cc. **21.** 10·36 g./cc. **22.** 17·62 g./cc. **24.** Zinc 23·22 % Copper 76·78%. **25.** One litre. **26.** Oil 10 gm. Water 37·5gm. **28.** (i) 1·1023lb. wt./in^2. 77·52. g. wt. cm.^{2} **29.** (i) 480 g.wt. (ii) 23520 dynes/cm.^{2} **30.** 12·6 wt /cm.^{2} **31.** 143 g.wt. cm.$^{-2}$ **32.** 2067·2 cm. **33.** 31·25 lb.wt./sq. in. **35** 28.8 ft. **37.** 20 g.wt. on the upper surface., 40.g. wt. on the lower & 30 g. wt. on the side face. **39** 2×10^9 g. wt. **41.** 105 lb. **43.** 0·97 g. cm.$^{-3}$ **44.** 0·87 **45.** 1·066 **46.** 0·9 **47.** 1·25 g.cm.$^{-3}$ **48.** 300 lb. wt. **49.** 100 : 1· **50.** 3960 lb. **51.** one cm. 0·5 ton. **53.** 28·8 lb. wt. wt. 727·3 ton. **54.** 0·8 ton. wt. 1·0 ton. wt. 600 strokes. **55.** 50 g. wt. **56.** 3 g. wt. **57.** (1) 10 gm. (2) 8 gm. **58.** 2·50, 0·87 **60.** 8·42, 0·88 **61.** 11·2, 0·8, 0·83, 1·04. **62.** 0·91, 1·26 **64.** 2·466. **67.** 0·26 **68.** 17.41 cm. **69** 7·9 gm. wt. **70.** 347·4 tons. **72.** (1) 2137 g. wt. (2) 4945 g wt. **73.** 28·44 ft./sec^2. 1 sec. **75.** 11·05 cc. **76.** 19 cc. **77.** 0·675 gm. **78.** 8 g. cm.$^{-3}$ **79.** 0·9 **80.** 0·7, 0·875 **81.** 0·07 cu. m. **82.** 12500 lb. **83.** 14 ft, 64000 lb. **84.** 0·525 of volume. **86.** 1·02 **87.** 7 cm. cube. **90.** 0·857 **91.** 1·166. **92.** 3.75 cm· **93.** 0·92 **94.** 1·26. **95.** 1·176 **97.** 0.91 g cm.$^{-3}$ **99.** 8011 metres. **100.** 53312 dynes- cm.$^{-2}$ **101.** 141 cm. of mercury. **102.** 12 ft. **105.** 16. 66 lb. wt., in^{-2} **107.** 357·5 kg. wt, **108.** 2000 cu. m. **109.** 55408 kg. wt. **111.** 1·8 atomspheres. **112.** 39270 cc. **113.** 12·5 lb.wt. in^{-2} **115.** 30 inches **116.** 73·48 cm. **117.** 70 cm. of mercury. **120.** 12 strockes. **121.** 26·77 m. **123.** 1·0624 g. cm.$^{-2}$ **125.** $2·5 \times$ original density. **127.** 6 : 1. **128.** 14·47 in. **129.** 38·97 cm. **130.** 30·6 ft. 34 ft.

MECHANICS

1. 27·3 mi/hr. **2.** 45 mi/hr. **3.** 45 mi/hr. **4.** 500 ft./min. **5.** 33°42′ to the north of west. 18·01 mi./hr. **6.** 75·47 mi./hr.

7. 1 furolng downstream. 8. 21·21 mi./hr. each. **10.** 6·01 ft **11.** 76·21 ft./sec. **12.** 10°18′ with the direction of the train. **14.** 3·6 ft./sec.^{2}. **15.** 6 sec. **17.** 45 mi./hr. **19.** 2·2 ft./sec.$^{-2}$. 15 ml./hr. 330 ft. **20.** 242 ft. **21.** 2800 cm. **23.** 2 ft /sec.^{2} 10 ft./sec: **25.** 32 ft. sec.$^{-2}$. **26.** 144 ft. **27.** 120 times./min. **28.** 48 ft above the ground; 48 ft. **29.** $\frac{1}{100}$ sec. **31.** 500 ft./sec. **33.** 1440 ft. **34.** 16 ft. per sec. **36.** (1) 528 ft. in advance; 4 sec. earlier. (2) 792 ft. in advance 6 sec. earlier. **37.** 594 ft. **38.** Path is parabola 5 sec. Horizontal, distance of 400 ft. Vertically downwards, **40.** 497·9 ft./sec. **41.** 160 ft.sec. 400 ft. **43.** 111·8 ft., 2·236 sec., 3·15 sec. **44.** 256 ft./sec. **45.** Horizontal, 443·4 ft. **46.** 20000 ft. **47.** 128·5 ft./sec. 85°6′ to the horizontal. **48.** 20 lb. ft./sec. **50.** 216 gm.cm./sec. **51.** 54 ft /sec. 2028 pdls. **52.** 2·5 ton wt. **53.** 2·7 lb. wt. **54.** 2500 pdls. **55.** 200 cm./sec. 18×10^5 dynes. **56.** 99 sec. **57.** 35 gm. **59.** $\frac{5}{8}$ ton. wt. **60.** 0·32 ft· sec.$^{-2}$, 19·2 ft./sec. **61.** 5625000 dynes. **62.** 257·1 ft. **63.** 12320 lb. wt. **64.** 49 kg. **66.** 2 sec. **67.** 2860 ft. **68.** 4 ft. sec.$^{-2}$, 32 ft. **69.** 500 ft., 100 ft./sec. **70.** 2·5 sec. **72.** 76·8 pdls, 25·6 ft., **74.** 10 lb., 12 lb., 320 pdls. 360 pdls. **76.** 3 oz. **77.** 122 g. 120100 dynes. **78.** 62 kg. Rope will move with acc. of 14 cm. sec.^{2}. **79.** 70 cm./sec. **80.** 3 sec. **82.** 500 m./sec. **83.** $1\frac{2}{3}$ ft./sec. **84.** 2 ft./sec., 4 ft./sec. **85.** $\frac{3}{4}$ ton wt. 75 lb. wt. **86.** 150 lb. wt. **87.** 90 lb. wt. **88.** 8 ft./sec.^{2}, 900 ft. **89.** 125 lb. **91.** 6·08 ft./sec.^{2}. Elevator descends. **92.** 18 lb.wt. **94.** 10 lb.wt. **96.** 216·5 lb. wt, 125 lb. wt. **97,** 15 lb.wt. **99.** 0·6 ton.wt. **100.** 20 lb.wt. **101.** 10 lb.wt. 11·55 lb.wt. **102.** 21·21/cosθ. **103.** 10 lb.wt. **104.** 104·4 lb. **105.** 30·02 lb. wt. **106.** 800 kg. wt. 692·8 kg. wt. **107.** 29·06 lb. wt. **108.** 1·154 lb. wt. **111.** 70 lb. each. **112.** 150 lb wt. at *B*., 450 lb.wt. at *A*. **113.** 4 ton wt. 4·5. 3·5 ton wt. **114.** 25 in. from *C*. **115.** 4·72 ft. from the man·

116. 0·440 lb. wt. 4·8 ft. **117**. 20 lb.wt. **118**. 60 lb. wt. 1920 pdl. ft. **119**. 30 lb. wt. **120**. 8 lb. wt., 48 lb. wt. **121**. 9600 lb. wt. **122**. 6 ft. **123**. 37° nearly. **125**. 50 lb. **126**. 16 gm. **129**. 0·15. **130**. 448 lb. wt. **131**. 20 lb.wt. **132**. 36 lb.wt. 2·90 lb.wt. **133**. 34·64, 0·3464. **134**. 0·295 *W*. **136**. 4 H.P. **137**. 0·14 H.P. **138**. 440 lb.wt. **139**. 10^8 dynes. **140**. 400 H. P. **142**. 3750 ft. lb. 0·682 H. P. **143**. $2{\cdot}413\times10^{11}$ ft. lb. **144**. 2000 H.P. **145**. $1{\cdot}419\times10^8$ ft. lbs. 429·8 H.P. **147**. 240 joules. **148**. 2000 ft. pdls., 2500 ft. **149**. 342 ft./sec. 1820 ft.171 lb.ft./sec.29241 ft.pdls. **151**. 5133·3 lb.wt. **152**. 300 poundals. **153**. 625×10^6 dyn. **154**. $12{\cdot}5\times10^9$ dyn. **156**. 24 ft. lbs. **157**. 96·04 joules 1·96 joules. **159**. 12 ft., –4. **160**. 98×10^4 dynes. **161**. 125 lb. wt. **162**. 280 lb.wt. **164**. 2 oz.wt. **165**. 20 lb. **166**. 150 lb., 5. **168**. 10 ft, **169**. 41·66 lb.wt. 38·46 lb.wt. **171**. 1·2 in. **173**. Loss of about 2 As.

LIGHT

1. 8 min. 20 sec. **2**. $5{\cdot}952\times10^{14}$ mi. **3**. 8·865 years. **5**. 125 ft, **6**. 6 ft. **9**. 36 candles **9*a***. 27 cm. to help 10 c.p. lamp. **11**. 25 : 32. **12**. (*a*) 2·8, 3·5 ft., in between : 14·5, 20·5 ft. one side. **14**. 54 : 625. **15**. Take the lamp nearer by 5·1 cm. **17**. 17 lamps. **18**. 355·7 c p. **19**. 2 ft. **20**. 100 sec. **21**. 6 ft. **22**. 55°. **24**. 22·5° 21·5′. **27**. 15 in. in front of ; 5 inches. **29**. 25′, 15′; in front of the mirror. **30**. 8″ × 6″, inverted, real magnified image. **32**. Virtual, $11\frac{7}{13}$ cm. behind the mirror, $\frac{6}{13}$ cm. **32*a***. 8 cm. $6\frac{2}{3}$ cm. $3\frac{1}{3}$ cm. **33**. 5·55 cm. **34**. 9·23 cm, **35**. 5 cm. behind the convex surface. Image is inverted and virtual. **36**. 1·3. **38**. 35·25′ **39**. 42°50′. **40**. 15° nearly. **42**. 2 in. **43**. 1.5. **44**. 1·525. **45**. 7·5 cm. The centre of the circle to lie on a vertical through the lamp. **46**. 3 cm.

47. 0·38 in. **48.** 1·414. **49.** 99°38′. **50,** 26°24′. **51.** 54°42′. **52.** 1·654, 37°15′. **53.** 1·532. **55.** 32·4′, 100′. **58.** 2·25 cm. from the surface to which it is nearest. **59,** 2·5 cm. from the surface. **61.** 2·87 in. **62.** 15·58 cm. **64.** 7·5 cm. **65.** 16 in. **67.** 12·5 cm. **68.** (i) 36·66 cm. from the side of the object (ii) real erect (iii) 6·0 inches. **69.** 10 cm. **70,** 15·8 cm. **71.** 1·33. **72.** $154\frac{2}{7}$ cm. **73.** 10 cm. ***73a.*** 115·4 cm. **74.** concave lens 60 cm. **75.** 12 cm. behind the lens, 6 cm. **76.** 10 cm **77.** 90 cm. **78.** (i) 22·1 cm. behind the lens. (ii) 11·4 cm. in front of the lens. **79.** 20 cm. **80.** 15 cm. **81.** 200 cm. convex. **83.** 200 cm. concave. **85.** – 65 cm. convex. **86.** 4 dioptres **87.** 15 in. concave. **88.** Concave, $11\frac{1}{9}$ ft. **89.** 10 ft. **91.** 3·5. **92.** 6. **93.** 4·75, 6·31 cm. **93*a*.** (*a*) 3, (*b*) 2. **95.** 10 ft. **97.** (*a*) 15, (*b*) 17. **98.** (*a*) 12, (*b*) 12·63. **99.** 1·62 in. **102.** (i) 5, (ii) **5·102** Image is at infinity.

HEAT

2. 41° *F*, – 4°*F*, – 459.4°*F*. **3.** 233°*A*, 93°*A*, 506°*A* 313°*A*. **4.** – 273°*C*, 27°*C*, – 10°*C*, 100°*C*. **5.** – 0·45°*F*. **6.** 0·05°*C*. **7.** 0·104°*C*. **9.** $-2\frac{2}{9}$°*C*, $31\frac{1}{9}$°*C*. **11.** 20°*C*. **12.** $16{\cdot}7\times10^{-6}$. **13.** $6{\cdot}97\times10^{-6}$. **15.** 100·6 cm. 100·95 cm. **16.** 114°*C*. **17.** 394°*C*. **18.** 0·0264 in. **20.** 76·0228 in. **21.** 30·023 in. **22.** 757·6°*C*. **24.** 7·035 cm. **26.** 200·304 sq.cm. **26*a*.** 6·00264 sq.ft. **27.** 500·33 cc. **28.** 2·4 cc. **29.** 8·886 g/cc. **31.** 0·57 gm. **32.** 9 cm. **33.** 1·328 cc. **35.** 195·05°*C*. **36.** 98·96°*C*. **38.** 60·352 cm. **39.** $18{\cdot}33\times10^{-5}$. **40.** 24 cc **41.** $18{\cdot}18\times10^{-5}$. **42.** $8{\cdot}25\times10^{-6}$. **44.** $8{\cdot}7\times10^{-6}$. **47.** 57°*C*. **48.** 61·9°*C*. **50.** 0·00356. **52.** 731·7 cc. **54.** 30°*C* fall, **55.** 1 litre. **57.** 174°*C*. **59.** 286·4 kgm. **60.** Vol. decrease of 230 cc. **61.** 562·5 mm. **62.** $948\frac{5}{6}$°*C*. **63.** 76 : 81. **65.** 1·986 cal./gm. mol/1°*C*. **68.** 0·0992 cal. **70.** 40°*C*.

71. 1542°*C*. **72**. 79·9 cal./gm. **74**. 240°*C*. **75**. 36·32 calories. **76**. $10{\cdot}16 \times 10^7$ cals. **77**. 43·6°*C*. **78*a***. 20 gm. 48·7 gm. **79**. 140 cals. **80**. 71·4 g. **81**. 0·11. **84**. 3660 calories. **85**. 537 kg. **86**. 2106 g. **88**. 61·47 cc. **89**. 0·0149 gm. **91**. 34·48%. **92**. 41.2%. **93**. 26 cc. increase. **94**. $3{\cdot}4 \times 10^4$ cc. **95**. 360 kg. **96**. 2·2306 g.

SOUND

2. 2296 ft. **3**. 2296 ft. **4**. 5 mi. from *A*. **5**. 7141 ft. **6**. 16320 ft./sec. 4896 ft. **7**. add 33 sec. **8**. 746.6 ft. **9**. 3.122 sec. **11**. 4·5 ft. **12**. 330. **13**. 56 ft. **14**. 80 metres **16**. 256. **18**. 20 per sec. **20**. 21·6 cm. **21**. 50 rev./3 sec. **23**. 400 vib/sec. **24**. 348·2 m/sec. **25**. Diam = $2\frac{1}{3}$ cm. 342 m./sec. **27**. 540·6. **29**. increase by $l/32$. **30**. shorter $l/51$. **31**. 105·3 cm.

MAGNETISM

1. 20 dynes. **2**. 25, −25 units. **3**. 0·5 dynes. **4**. attractive force of one dyne. **6**. 4 cm. 1·78 dynes 8 units. **7**. 784 units **8**. 23·3 dynes. **11**. 18 dynes. **12**. 800 units. **13**. 0·125 gauss. **15**. 675 uits, **16**. 0·13 gauss. **18**. 0·1431 gauss. **19**. 0.035 *or* 0·19, 0·0312 dynes. **19*a***. $M_A : M_B = 9 : 7$. **20**. 0·01 gauss. **22**. 0·5 gauss. **23**. 123 dyne. cm. **24**. 1 : 2. **26**. 14·641 ergs. **28**. 150°. **29**. 1 : 2. **31**. 4995 pole. cm. **32**. 462 pole. cm. 260 pole. cm. **34**. 466·5 pole. cm. **35**. 41·5 cm. 2766 pole. cm. **36**. 27 : 64. **37**. 1 : 2. **38**. 3 : 5. **39**. 2 : 3.

STATIC ELECTRICITY

2. 8 dynes/unit charge. **3**. $\sqrt{\frac{3}{625}}$ dynes per charge along the bisector of $\angle BAC$. 4. 5·38 dynes. **5**. 11·64 dynes along *DB*. **8**. 9·9 E.S.U. **9**. 19.6 E. S. U. **11**. 29400 dynes. **12**. σ increase by 26 % **15**. 64·6 units. **16**. 8

units. 17. 150 units. 18, $-$ 1625 ergs. 19 $4\cdot77\times10^{-10}$ E. S. U. 20. 14 units. 3.32 ergs. 21. $10\cdot29\times10^{9}$ cm. per sec. 23. 10 cm. 25. 40 units. 26. 33·33, 16·66 E.S.U. 3·33units 27. 10 units. 100, 25. E. S. U. 8·33 units. 28. 2·5 cm. 30. 6:1 32. 2 mm. 33. 0.2 mm. 35. 8.0. 36. 462. cm. 37. 10πdynes. $5000/\pi$ cm. 4πdynes. $12500/\pi$ cm. 39. 1·09 cm. 40. 400. cm. 41. 2400, 400·800, 1200 E. S. U. 42. 100 E. S. U. 100 E.S.U. 200 E.S.U.

CURRENT ELECTRICITY

2. $5\cdot508\times10^{6}$ dynes. 3. 3 dynes. 4. 0.7 oersted. 5. 200 ergs. 7. 18 dynes. cm. 8. 6928 dynes. cm 9. 75π dynes cm. 11. 18·09 dynes cm. 12. 366 ergs. 14. 2·5 amps. 16. 0·48 gauss. 17. 10 amps. 18. 3·142 cm. 19. 49 turns. 21. 5 amps. 22. (*a*) 1.839 gauss. (*b*) 1·609 gauss. 23. (*a*) 1·303 gauss. (*b*) zero. 24*a*. $n=3.266$; 1·626; 2·708; 0·8166 N keeps north. 25. (i) 0·2122 gauss. (ii) One second. 26. 0·003 e. g. s. 27. 5·774 amps. 29. 120 coulombs. 30. $0\cdot1\pi$ gauss. 32. (*a*) 2π gauss. (*b*) 0·5774 to 2·7475 amps. 33. 0·358 gauss. 45°. 34. 0·625 amps. 36. $0\cdot1\pi$ gauss. 38. 0·25 amps. 38*a*. 0·1 amp. 40. 220 ohms. 41. 1·25 amps. 42. $\frac{1}{10}$ ampere. 43*a*. 344 ohmps. 44. 1 volt. 45. 73·33 *v*. 146·66 *v*. 46. 0·7333 amp. 46*a*. $I=\frac{12}{13}$ amp. $I_3=\frac{6}{13}$ amp. $I_9=\frac{3}{13}$ amp. 47. 1·15 ohms. 3·11 ohms. 48. 55 volts. 49. $2\frac{8}{11}$, $1\frac{4}{11}$, $\frac{10}{11}$ amps. 51. 20·5 ohms. 53. 2 ohms 54. 4 ohms. 56. 0·018 ohms. 57. 8 ohms. 2 ohms. 60. 255·6 volts 61. 556 *v*. 555*v*. 62. 0·5774 : 1. 65. 0·4 volts. 66. 13 ohms. 67. 50 ohms. 68. 6 ohms. 69. 20 ohms.

MISCELLANEOUS EXAMPLES

1. 12° to the south of east. 169 mi./hr. 2. $5\frac{2}{3}$°. 3. 4 ft., 1 sec. 4. 372·7 ft./sec². 5. 70. pdls. 32 ft. per sec².

6. (*a*) 20 lb.wt. (*b*) 20·62 lb. wt. **7.** 185 lb. wt. vertically upwards. **8.** 480 cm. **9,** 4410 m. 30 sec. 23520 m. **10.** 14·2′. **11.** 98 dynes. **12.** 354 cm./sec. **13.** 1·73 lb. wt, 3·46 lb. wt., **14.** 8·5 lb. wt. **15.** The man pulls himself up with an acc. of 28 cm. / sec^2. **16.** 1034 cm. **17.** 8 strokes **18.** 0·0012 sq. cm. **19.** 0.0005706. **20.** 55 lb/in^2. **21.** 150°C. **22.** 866 cc. **23.** 28 per minute. **24.** 6·25 min., 40·5 min. **25.** 997·2 cals **26.** 0·515 **27.** 9 gm. **28.** 600 gm. **29.** 5·35 min. **30.** 40°C **31.** 11$_5$/° **32.** 10°C **33.** 0·5 ft. **34.** 4/3 **35.** 9 in. **36.** 5ft, 3·75 ft. **37.** 14 cm., 12 cm. **38.** 20°35′. **39.** 41°20′ **40.** $-$19·35 cm., convex lens. **41.** 24 cm., Image wil be (1) nearer by $3\frac{1}{3}$ cm. (2) farther by 80 cm. **42.** 12 cm. at infinity. **43.** short sight, $13\frac{7}{11}$ cm. concave lens. **44.** (*a*) Concave $f=300$ cm. (*b*) convex, $f=37\frac{1}{2}$ cm. **45.** 150 cm. from stronger one. **46.** 30·56%. **47.** 12·6 cm. 10 cm. **48.** $\frac{2}{3}$ units towards. S pole. **49.** 692·8 pole-cm: **50.** 2·4 dynes cm. **51.** $M_A : M_B = 1:2$ **52.** 527 : 1000 **53.** 857.4 **54.** 19·6 *e. st. u.* **55.** 1×10^{-6} cm. / sec.^{2} **56.** 9·808 dynes 980·8 cm. /sec^2. **57.** 2×10^6 units **58.** 0·0318 and 0.159 per cm.^{2} **59.** From smaller to larger one. 0·563 of the total Q on larger one. 0·437 of Q on smaller one. **60.** $A = 100$ cm. $B = 33·3$ cm. 250 units flow to B; $V = 7·5$. **61.** 3.2×10^{-18} dynes. **62.** 3000 cm. **63.** 20 cm.

FUNCTIONS OF ANGLES AT 1° INTERVALS.

Angle.	Radians.	Chords.	Sine.	Tangent.	Cotangent.	Cosine.			
0°	0	·0	0	0	∞	1	1·414	1·5708	**90°**
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	**89**
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	**88**
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	**87**
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	**86**
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	**85**
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	**84**
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	**83**
8	·1396	·139	·1392	·1405	7·1154	·9903	1·312	1·4312	**82**
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	**81**
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	**80**
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	**79**
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	**78**
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	**77**
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	**76**
15	·2618	·261	·2588	·2679	3·7321	·9659	1·217	1·3090	**75**
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	**74**
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	**73**
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	**72**
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	**71**
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	**70**
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	**69**
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	**68**
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	**67**
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	**66**
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	**65**
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	**64**
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	**63**
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	**62**
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	**61**
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	**60**
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	**59**
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	**58**
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	**57**
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	**56**
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	**55**
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	**54**
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	**53**
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	**52**
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	**51**
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	**50**
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	**49**
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	**48**
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	**47**
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	**46**
45	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	**45**
			Cosine.	Cotangent.	Tangent.	Sine.	Chords.	Radians.	Angle.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3777	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3175	3483	3191	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3505	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3081	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	[illegible]	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	[illegible]	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	[illegible]	[illegible]	[illegible]	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	[illegible]	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	[illegible]	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

RECIPROCALS OF NUMBERS. FROM 1 TO 10.

[*Numbers in difference columns to be subtracted, not added.*]

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
1·0	1·000	9901	9804	9709	9615	9524	9434	9346	9259	9174									
1·1	·9091	9009	8929	8850	8772	8696	8621	8547	8475	8403									
1·2	·8333	8264	8197	8130	8065	8000	7937	7874	7813	7752									
1·3	·7692	7634	7576	7519	7463	7407	7353	7299	7246	7194									
1·4	·7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38	43
1·5	·6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33	38
1·6	·6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	20	29	33
1·7	·5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	20	23	26	29
1·8	·5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23	26
1·9	·5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21	24
2·0	·5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
2·1	·4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17	20
2·2	·4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
2·3	·4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14	16
2·4	·4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
2·5	·4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	14
2·6	·3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	8	10	11	13
2·7	·3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
2·8	·3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
2·9	·3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
3·0	·3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	7	9	10
3·1	·3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
3·2	·3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
3·3	·3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
3·4	·2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
3·5	·2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
3·6	·2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6	7
3·7	·2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6	6
3·8	·2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
3·9	·2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5	6
4·0	·2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
4·1	·2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5	5
4·2	·2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
4·3	·2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
4·4	·2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
4·5	·2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4	4
4·6	·2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
4·7	·2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
4·8	·2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3	4
4·9	·2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
5·0	·2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
5·1	·1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0	1	1	2	2	2	3	3	3
5·2	·1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
5·3	·1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
5·4	·1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3

RECIPROCALS OF NUMBERS. FROM 1 TO 10.

[Numbers in difference columns to be subtracted, not added.]

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
5·5	·1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
5·6	·1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
5·7	·1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
5·8	·1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	1	1	1	1	2	2	2	3
5·9	·1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
6·0	·1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
6·1	·1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
6·2	·1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
6·3	·1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
6·4	·1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
6·5	·1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
6·6	·1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
6·7	·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
6·8	·1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
6·9	·1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
7·0	·1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
7·1	·1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
7·2	·1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
7·3	·1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
7·4	·1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
7·5	·1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
7·6	·1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
7·7	·1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
7·8	·1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
7·9	·1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
8·0	·1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
8·1	·1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
8·2	·1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
8·3	·1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
8·4	·1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
8·5	·1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
8·6	·1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
8·7	·1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
8·8	·1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
8·9	·1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
9·0	·1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
9·1	·1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
9·2	·1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
9·3	·1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
9·4	·1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
9·5	·1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
9·6	·1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
9·7	·1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
9·8	·1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
9·9	·1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1

NATURAL TANGENTS.

Degrees.	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences. 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0′ 0°·0	6′ 0°·1	12′ 0°·2	18′ 0°·3	24′ 0°·4	30′ 0°·5	36′ 0°·6	42′ 0°·7	48′ 0°·8	54′ 0°·9	Mean Differences 1	2	3	4	5
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2·0057	2·0145	2·0233	2·0323	2·0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3·0061	3·0237	3·0415	3·0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970	Mean differences cease to be sufficiently accurate.				
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞			—											